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Tithinirṇaya: A Calendrical Text of the Mādhva Tradition for Religious Observations

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1 INTRODUCTION

THE TITHINIRṆAYA (DETERMINATION OF THE *TITHI*) is an astronomical *karaṇa* text.[1] Its epoch is April 3, 1308 CE. It consists of twenty-eight verses that give the procedure to compute the calendrical element known as *tithi* (lunar day),[2] at sunrise on a desired day, for an observer approximately located at a latitude of 12.78°.[3] The text follows the Haridatta's *parahita*[4] corrected Āryabhaṭa system, which is evident from the *dhruvas*,[5] (22) and (33), of the Moon and Moon's apogee, respectively, at the epoch. The primary application of this text, as evident from its invocatory and concluding verses, is to precisely determine the days, devoted to lord Viṣṇu, on which a fasting ritual is to be observed. The text appears to be intended for the followers of the Mādhva tradition,[6] as evidenced by its

1 A genre of astronomical texts which chooses a recent epoch and dictates a simpler procedure in computing the aspects of astronomy, i.e., calendrical elements, eclipses, etc., without presenting the rationale involved in the computations.

2 A time unit in which the longitudinal separation between the Moon and the Sun increases by 12°.

3 The latitude corresponds to the location of the author, proposed to be Śrī Trivikramapaṇḍitācārya in Section 1.3.

4 A system proposed to correct the longitudes of the planets, computed from *Āryabhaṭīya* astronomical parameters, post *śaka* 444 or *kali* year 3623.

5 The fixed mean longitudes proposed by the author at the epoch.

6 The followers of Śrī Madhvācārya, the chief proponent of *Dvaita* school of *Vedānta* philosophy. It may be noted that currently, within the Mādhva tradition, only the *maṭhas* (religious establishments in the lineage) like Sode, Kṛṣṇāpura, Śīrūru, Kāṇiyūru, and Bhīmanakaṭṭe subscribe to the *Tithinirṇaya* method of calendrical computations.

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usage exclusively within that community today.[7]

The first verse of the text is an invocation to Viṣṇu, which lays out the purpose of the text. Verses 2–24 prescribe the procedure to compute the *tithi* by finding the true longitudes of the Sun and the Moon at true sunrise. Verses 25–28 discuss rules with regards to the observation and breaking of the fast on various days.

1.1 PRIOR PUBLICATIONS AND AVAILABILITY OF MANUSCRIPTS OF *TITHINIRṆAYA* AND ITS COMMENTARY

A perusal of manuscript catalogs and publication database reveals that there are several texts named *Tithinirṇaya*.[8] It appears to be a popular name adopted by scholars who wish to discuss the *vratas* (holy practices) associated with the *tithis* across different lunar months in a year. Though the *Tithinirṇaya*, which is the subject of this work also discusses, in brief, the *vratas* like *ekādaśī* and *Viṣṇupañcaka*, a large portion of the work is dedicated to the computation of *tithi*, which sets it apart from the other published *Tithinirṇayas*.[9]

The current *Tithinirṇaya* was first brought to light in 1974 by Padmaśrī[10] Bannañje Govindācārya in his critical edition of the *Sarvamūlagranthas*.[11]

He records[12] that the manuscripts of the text were available in various Mādhva *maṭhas* (religious establishments in the lineage),[13] and his edition was based

7 Usually, the Mādhvas fast two days a month, corresponding to the *ekādaśī tithis*. Optionally, they may also fast on the days corresponding to the *amāvāsyā* and *pūrṇimā tithis*, as well as the days corresponding to the *Śravaṇā-nakṣatra*. Thus, in all, they may fast from two to five days in a lunar month.

8 See pages 170–171 in volume 8 of New Catalogus Catalogorum at `https://vmlt.in/ncc/8?page=90`, and manuscripts of Allahabad Museum at `https://indianculture.gov.in/manuscripts?search_api_fulltext=tithinirnaya&`.

9 See Śāstri (1940), and Ś. R. Jhā (1983). Even within the Mādhva tradition, there appears to be another text named *Tithinirṇaya*, attributed to Ānandatīrthācārya (son of Tāmraparṇī Viṭṭalācārya), which focuses solely on the performance of the *vratas* and does not contain astronomy.

10 A highly regarded civilian honour of the Republic of India.

11 The *Sarvamūlagranthas* generally refer to the collection of 37 works attributed to Śrī Madhvācārya. This critical edition of Bannañje (1974*b*) contains 39 works, including *Tithinirṇaya*. Prior to the publication of the *Tithinirṇaya* as part of this collection in 1974, the Mādhva *maṭhas* in Udupi used to construct calendars as per the *Āryabhaṭīya Vākyakaraṇa* (this is the name given in the *pañcāṅga*. The calender makers have confirmed this to be the *Karaṇaprakāśa*). Following Bannañje Govindācārya's attribution of the *Tithinirṇaya* to Śrī Madhvācārya, some of the *maṭhas* adopted this text for their calendrical computations. This adoption was perhaps made easier by the fact that the *Tithinirṇaya* produced the same results as the *Āryabhaṭīya Vākyakaraṇa*.

12 Bannañje (1974*b*: 175) says 'सम्प्रति तु बहुषु स्थलेष्वन्यान्यमठेषु चास्य हस्तलिखितानि पुस्तकान्युपलब्धानि।'

13 Śrī Madhvācārya anointed nine disciples as his successors, from which nine *maṭhas* got established. Eight out of the nine, Palimāru, Adamāru, Kṛṣṇāpura, Puttige, Śīrūru, Sode, Kāṇiyūru, and Pejāvara, based in Udupi are known as *aṣṭamaṭhas* (eight re-

on the manuscript found in the archives of the Pejāvara *maṭha*, Udupi.[14] Having said that, while providing the Sanskrit commentary and working out an example, he also mentions the scribal errors found in other available manuscripts without revealing their location. The scribal errors or the alternate readings captured by Bannañje Govindācārya are provided in the respective sections.

A Kannada translation of the text, which appears to have been at least partly inspired by Bannañje Govindācārya's Sanskrit edition, was carried out by Vyāsadāsa (2007). Here too, we find mention of the availability of the manuscripts in Pejāvara, Sode and other *maṭha*s.[15] This edition also contains a brief commentary and works out an example.

Unfortunately, neither of these publications provides cataloging details pertaining to the manuscripts. We were unable to trace them in the archives of Pejāvara, Sode, Kṛṣṇāpura, and Kāṇiyūru *maṭhas* in Udupi, Subrahmaṇya *maṭha* in Subrahmaṇya village, the Vyāsa Madhva Sevā Pratiṣṭhāna in Bangalore, and Viśva Madhva Mahā Pariṣad of Uttarādi *maṭha* in Bangalore. Some of the other *maṭha*s either do not maintain archives, or we were unable to access them.

However, we have been able to access a facsimile of a manuscript, which appears to be from a private collection, and contains a commentary by Madhusūdana Bhikṣu,[16] a seventeenth or eighteenth century monk.[17] A preliminary analysis reveals the manuscript to be incomplete, with missing verses and omissions by the scribe in the commentary. Further, the last four verses 25–28 of *Tithinirṇaya* are not discussed in this commentary; instead, the commentator explains the procedure to obtain the other calendrical elements like *nakṣatra*, *yoga*, and *karaṇa*.

Moreover, the commentator appears to adopt a novel approach to interpreting the text and explains that he has made necessary emendations due to the errors that have crept in in the available manuscripts and their scarcity during

ligious establishments). The ninth is the source of *maṭha*s like Uttarādi, Vyāsarāja, and Rāghavendra. These *maṭha*s are prominent in the lineage.

14 Bannañje (1974*b*: 175) says 'ग्रन्थोऽयमेकस्मिन् मूलकोशावलम्बेन लिखिते श्रीपेजावरमठीये प्राचीनकोशे उपलब्धः।'

15 Nāgabhūṣaṇa Rao, in the foreword, says 'ಇದರ ಹಸ್ತಲಿಖಿತ ಪ್ರತಿಗಳು ಶ್ರೀಪೇಜಾವರಮಠ, ಶ್ರೀಸೋದೆ-ವಾದಿರಾಜಮಠ ಮತ್ತು ಇತರ ಕೆಲವು ಮಠಗಳಲ್ಲಿಯೂ ಉಪಲಬ್ಧವಿದೆ.' See Vyāsadāsa (2007: iii).

16 Rāmanāthācārya (1996) is the first to report the details of this manuscript in the January edition of the Tatvavāda. From Rāmanāthācārya (1996), Viṣṇudāsa (2014: 144), and Bhikṣu (n.d.), we learn that Madhusūdana Bhikṣu was a disciple of Śrī Satyapūrṇatīrtha, and Śrī Satyavijayatīrtha, the 22nd and 23rd pontiffs of Uttarādi-*maṭha*, respectively.

17 Rāmanāthācārya (1996: 37) places him in the seventeenth century. B. N. K. Sharma (1981: 209) places Madhusūdana Bhikṣu's preceptors, Śrī Satyapūrṇatīrtha, and Śrī Satyavijayatīrtha, in the eighteenth century, while Dasgupta (1949: 56) places them in the seventeenth century.

his times.[18] Hence, we refer to this commentary only sparingly in our discussion, wherever it aligns with our understanding of the text.

Therefore, since the source manuscripts of the *Tithinirṇaya* are unavailable, this work derives its verses from the main reading found in the published works of Bannañje (1974*b*) and Vyāsadāsa (2007).

1.2 DATE OF COMPOSITION

The second verse of the *Tithinirṇaya* encodes the epoch of the text in the phrase *bhūśrībhinnākicintya*, employing the *kaṭapayādi* system. Decoded, this phrase corresponds to the number 1610424, which gives the *kali-ahargaṇa* or, the number of days elapsed since the beginning of the *kaliyuga*. This day corresponds to the beginning of the true sidereal year (*Meṣa-saṅkrānti*), when 4409 years have elapsed in the *kaliyuga* calendar or April 3, 1308 in the Gregorian calendar. This would also correspond to *Caitra-śukla-caturthī* in the *śaka* 1230 (elapsed), named *Kīlaka*.

Based on the nature of the text and epoch chosen, the astronomical texts are classified into *siddhānta*, *mahātantra*, *tantra*, and *karaṇa*.[19] As *Tithinirṇaya* employs a relatively recent epoch and outlines a simplified procedure to compute the *tithi* without presenting the complete theoretical framework, it belongs to the *karaṇa* category.

1.3 AUTHORSHIP

The *Tithinirṇaya*'s source text lacks any information about its authorship. Nevertheless, Bhikṣu (n.d.), Bannañje (1974*b*) and Vyāsadāsa (2007) assert the author to be Śrī Madhvācārya. However, many authoritative texts within the Mādhva tradition do not record *Tithinirṇaya* among the works authored by Śrī Madhvācārya. Certain earlier works by eminent saints also contain statements that would contradict such an attribution to Śrī Madhvācārya. Alternatively, some scholars propose the author to be Śrī Trivikramapaṇḍitācārya, a direct disciple of Śrī Madhvācārya. In the subsequent discussion, we present these varying perspectives chronologically.

The earliest texts within the Mādhva tradition do not include the *Tithinirṇaya* among the works attributed to Śrī Madhvācārya. The *Sumadhvavijaya*, recognized as an authentic life sketch of Śrī Madhvācārya, authored by his near contemporary Śrī Nārāyaṇapaṇḍitācārya in late thirteenth century, while discussing the works of Śrī Madhvācārya does not mention the *Tithinirṇaya* or any other work

18 It is worth noting that there was a scarcity of *Tithinirṇaya* manuscripts during Madhusūdana Bhikṣu's period (seventeenth or eighteenth century CE), whereas it was otherwise during the time of Bannañje (1974*b*).

19 See *Vākyakaraṇa*, Sastri and Sarma (1962: 7), which states तत्र सिद्धान्त-महातन्त्र-तन्त्र-करणभेदेन गणितस्कन्धस्य चतुर्विधत्वम्।

of that genre.[20] Furthermore, subsequent commentaries on *Sumadhvavijaya* also do not include *Tithinirṇaya* in their enumeration of Śrī Madhvācārya's works. In other compositions providing enumerations of Śrī Madhvācārya's works, such as *Granthamālikā* by Śrī Vyāsarājatīrtha (1478–1539), *Pūrṇaprajñagranthamālikā* by Śrī Yadupatyācārya (1580–1630), and another work of the same name by Bidarahaḷḷi Śrī Śrīnivāsācārya (1600–1660), the *Tithinirṇaya* is neither mentioned by name nor categorized by genre.[21]

Further, Śrī Vādirājatīrtha (1480–1600), the twentieth pontiff of the Sode *maṭha*, in his *Ekādaśī-nirṇaya*, categorically states that Śrī Madhvācārya did not author any work dealing with the classification of *viddhaikādaśī*. Since this classification is dealt with in verse 25 of *Tithinirṇaya*,[22] it can be inferred from Śrī Vādirājatīrtha's statement[23] that Śrī Madhvācārya did not author the *Tithinirṇaya*.

Moreover, many scholars of the tradition, starting from Śrī Vādirājatīrtha, attribute the same verse to Śrī Trivikramapaṇḍitācārya.[24] Śrī Tāmraparṇī Śrīnivāsācārya, in his commentary of Śrī Madhvācārya's *Kṛṣṇāmṛtamahārṇava*, mentions the source of this verse to be Śrī Trivikramapaṇḍitācārya's *Tithinirṇaya* and also states Śrī Vādirājatīrtha's *Ekādaśī-nirṇaya* to be a commentary of this *Tithinirṇaya*.[25] Going by these statements of considerable authority, it seems that the author of the *Tithinirṇaya* is likely to be Śrī Trivikramapaṇḍitācārya.

On the other hand, the attribution of the work to Śrī Madhvācārya is fairly recent. The earliest such attribution is made by Madhusūdana Bhikṣu (c. seventeenth century CE), in his commentary of *Tithinirṇaya*,[26] where he states that the work was composed by Śrī Madhvācārya before he became a monk.[27] Further, Bannañje (1974*b*), independently ascribes the text to Śrī Madhvācārya. He refers to a statement[28] from an unspecified ancient text on *tithi* available in Palimāru *maṭha*, which attributes the *Tithinirṇaya* to an *ācārya*. Interpreting *ācārya* as

20 See *Sumadhvavijaya* verses XV.73–90, Shyamachar and Pandurangi (2001: 403–413).

21 See Shyamachar and Pandurangi (2001: 495–496).

22 See Section 16.

23 See *Ekādaśī-nirṇaya* verse 8(a,b), B. P. N. Rao (1994: 26), which states शिष्यायोपदिशत् ग्रन्थे न बबन्ध सदाग्रणीः।

24 See *Ekādaśī-nirṇaya* verse 30, B. P. N. Rao (1994: 34), *Smṛtimuktāvalī*, Giri Ācārya (2016: 147–148), *Karmasiddhānta*, Rāmanāthācārya (2013: 93).

25 See Karaṇam and Vādirājācārya (2002: 183).

26 See Rāmanāthācārya (1996) and Bhikṣu (n.d.) where, in the introduction, the colophon states: आनन्दतीर्थमुखान्निसृतस्तिथिनिर्णयः। तस्य व्याख्यां यथाबोधं करिष्ये तत्कृपाबलात्। and in the end, colophon states: इति श्रीमदानन्दतीर्थार्यमुखनिसृतः तिथिनिर्णयनामा यः तस्य व्याख्या कृता मया। The name Ānandatīrtha, mentioned here, was given to Śrī Madhvācārya by Acyutaprekṣatīrtha when he was crowned as the ruler of the Empire of *Vedānta*. See *Sumadhvavijaya* verses V.1–2, Shyamachar and Pandurangi (2000: 201–202).

27 See folio 1, where Bhikṣu (n.d.) mentions …सन्यासग्रहणात्पूर्वमेव तिथिनिर्णयाख्यं ग्रन्थं कर्तुकामाः…।

28 Bannañje (1974*b*: 175) mentions आचार्यैस्तथैव तिथिनिर्ण्णयेऽभिहितमित्यार्येणैव विनिर्ण्णयः।

Śrī Madhvācārya and considering the prevalence of the manuscripts of this text primarily in the Mādhva *maṭhas*, Govindācārya attributes the work to Śrī Madhvācārya. He also claims that it was composed when Śrī Madhvācārya was around 70 years old, which contradicts the statement of Madhusūdana Bhikṣu.[29] Further, Vyāsadāsa (2007), simply accepts the claim made by Govindācārya and attributes the text to Śrī Madhvācārya.

In conclusion, considering the absence of the *Tithinirṇaya* among the works attributed to Śrī Madhvācārya by some of the earliest and most prominent scholars of the Mādhva tradition, we find it difficult to accept the attribution of this text to him by the recent scholars. In light of Śrī Vādirājatīrtha's statement that Śrī Madhvācārya never composed any work on the classification of *viddhaikādaśī*, the presence of verse 25 in the *Tithinirṇaya*, which deals with this very subject matter, further reduces the likelihood of his authorship of this text. On the other hand, the attribution of this very verse, by several scholars, to Śrī Trivikramapaṇḍitācārya, and the proximity of his home town Kāvu ($\phi = 12.53°$)[30] to the latitude ($\phi = 12.78°$) employed for *cara* computations in this text,[31] lead us to believe that Śrī Trivikramapaṇḍitācārya may perhaps be a more probable candidate for the authorship of the *Tithinirṇaya*.

1.4 CONTENTS OF THE TEXT

In this work, the verses of *Tithinirṇaya* are grouped across different sections based on their content, as shown in Table 1. Section 2 deals with the invocation, Sections 3–15 explain the procedure to compute *tithi*, and Sections 16–19 give the rules for observing the fast.

1.5 OVERVIEW OF THE PROCEDURE TO FIND *TITHI*

The Indian calendar, known as *pañcāṅga* (five limbs), primarily comprises five elements: *tithi*, *vāra*, *nakṣatra*, *yoga*, and *karaṇa*.[32] The *Tithinirṇaya* deals with the procedure to compute a *tithi*, which provides the time for undertaking Vedic rituals, *ekādaśī* fasts, etc. The computation of the *tithi* at any instant depends on the true longitudes of the Sun and the Moon at that instant. Generally, in calendar making, computations are carried out for the instant of sunrise at the observer's location. Thus, to determine the *tithi* at sunrise, the true longitudes of the Sun (θ_s^t) and the Moon (θ_m^t) have to be computed for that instant. To compute these, the general procedure laid out in the astronomical texts involves first

29 According the Mādhva tradition, Śrī Madhvācārya was ordained as a monk at a young age.

30 B. N. K. Sharma (1981: 213) mentions the ancestral house of Śrī Trivikramapaṇḍitācārya to be at Kāvu, Kāsargoḍ, Kerala.

31 See Section 14.1.2.

32 See S. B. Rao (2000: 64–70) for more details.

Section	*Verses*	*Content*
2	1	Invocation
Procedure to compute *tithi*		
3	2–3	Mean longitude of the Sun at mean sunrise at Laṅkā
4	4	Mean longitude of the Moon at mean sunrise at Laṅkā
5	5	Mean longitude of the Moon's apogee at mean sunrise at Laṅkā
6	6–7	*Deśāntara* correction: to obtain mean longitudes at mean sunrise at the observer's meridian
7	8–9	Sun's apogee and *bhujāntara* correction: to obtain mean longitudes at true sunrise at the observer's meridian
8	10–12	Rsine values of 24 arcs
9	13	Interpolation formula for obtaining the desired Rsine
10	14	Quadrants of Ecliptic and *bhuja*
11	15	*Manda* correction: to obtain true longitudes at true sunrise at the observer's meridian
12	16–18	Trepidation of the Equinox
14[a]	19–22	*Caradala* correction: for an observer's latitude of 12.78°
15	23–24	Elapsed *tithi* and the elapsed time in the current *tithi*
Rules for observing the fast		
16	25	Determining *viddhaikādaśī*
17	26	Fasting days of *Viṣṇupañcaka vrata*
18	27	Reaping the full benefits of a fast
19	28	*Saṅkoca-dvādaśī* or *Sādhana-dvādaśī*

[a] Section 13 discusses ignoring the *udayāntara* correction, which accounts for the obliquity of the ecliptic, in *Tithinirṇaya*.

Table 1: Contents of *Tithinirṇaya*.

computing the mean longitudes of the Sun (θ_s°) and the Moon (θ_m°) at the instant (t°) of mean sunrise for an observer at Laṅkā,[33] followed by a series of corrections i.e., *deśāntara*, *bhujāntara*, *manda*, *udayāntara* and *cara*. The algorithm depicting the series of corrections, along with brief rationales, is shown in Figure 1.[34]

33 Laṅkā is the point of intersection of the prime meridian (a meridian passing through Ujjayinī, Svāmīnagara, etc.) and the equator.

34 The notations employed in this work and their interpretations are summarized in Section 1.6.3.

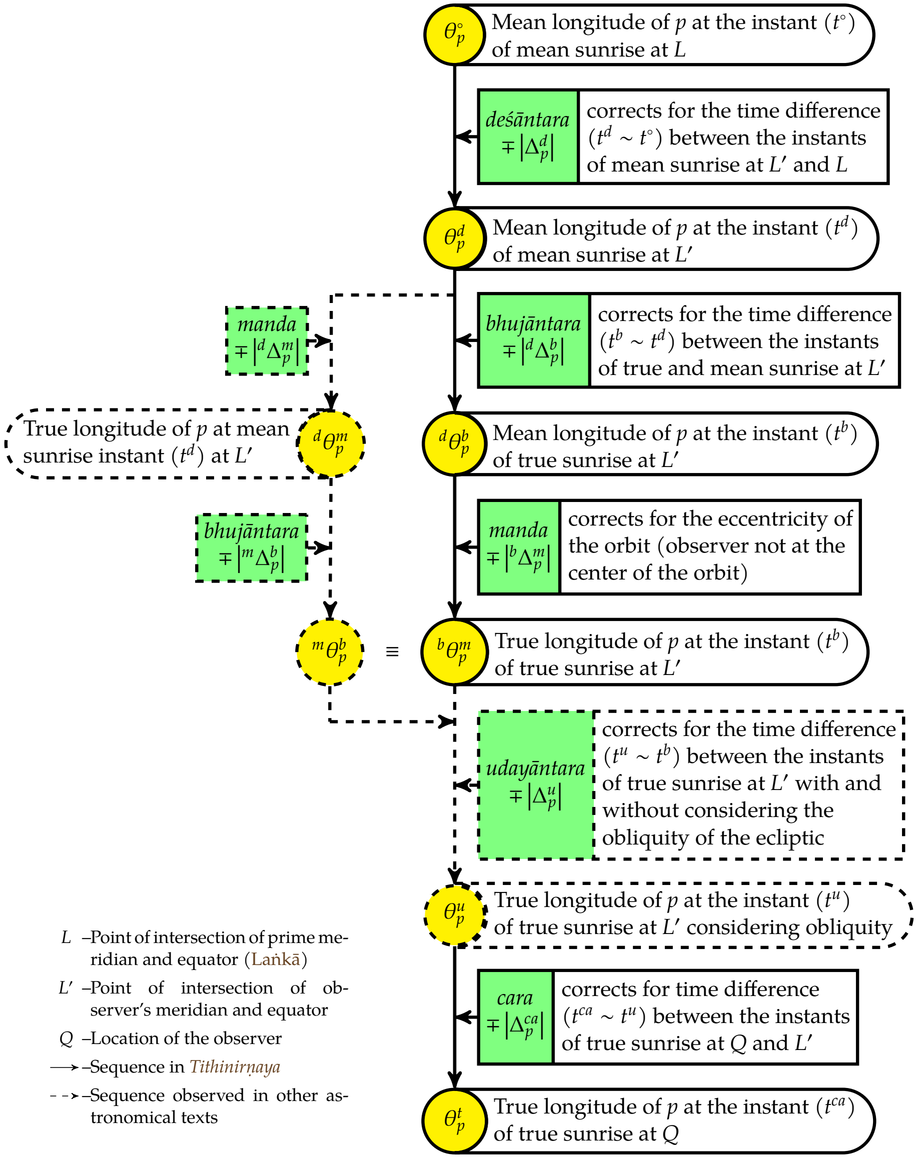

Figure 1: A diagram showing the sequence of corrections applied for obtaining the true longitude (θ_p^t) of a celestial body (p). Here, the subscript p can be replaced with s and m for the Sun and the Moon respectively.

To appreciate the physical significance of these corrections, let us consider Figure 2, which depicts a spherical Earth, its poles, the prime meridian,[35] and

35 See *Karaṇaratna* verse I.30, Shukla (1979: 21–22), which states that the prime meridian is the meridian passing through Ujjayinī, Svāmīnagara, etc.

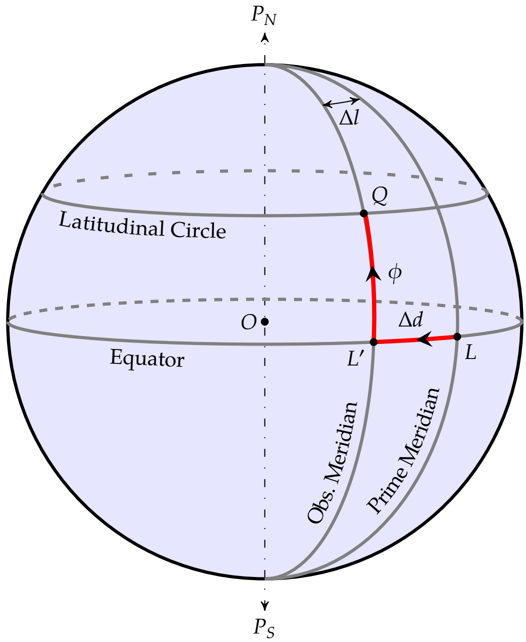

Figure 2: A diagram showing the locations, L, L' and Q, on the spherical Earth corresponding to which at their (mean or true) sunrise, the longitudes of the Sun and the Moon are computed.

the observer's meridian. Let L be the point of intersection of the prime meridian and equator, denominated as Laṅkā in Indian astronomical tradition. Let the observer be located at Q at a latitude $QL' = \phi$, on a meridian at a distance of $LL' = \Delta d$ *yojanas*[36] (along the equator, east or west) from the prime meridian. The sequence of corrections in Figure 1 takes us along the path $L \rightarrow L' \rightarrow Q$ highlighted in Figure 2, resulting in the conversion of mean longitudes at mean sunrise at L into true longitudes at true sunrise at Q.

To elaborate, the procedure commences with computing the mean longitudes of the Sun (θ_s°) and the Moon (θ_m°) or, in general, θ_p°, at the instant (t°) of mean sunrise at a reference location, typically taken to be Laṅkā (L) in Indian astronomy. Applying the *deśāntara* correction ($\mp|\Delta_p^d|$) accounts for the time difference ($t^d \sim t^\circ$) between the instants of mean sunrise at L' and L, thus obtaining their corresponding mean longitudes (θ_p^d) at the instant (t^d) of mean sunrise at L'. Next, the *bhujāntara* and *manda* corrections can be applied interchangeably. The *manda* correction (equation of center) considers the effect of the observer being away from the center of the orbit, and *bhujāntara* correction accounts for

36 A unit of length used by Indian astronomers.

Description	*Notation*	*Value*
Number of revolutions of the Sun	R_s	4320000
Number of revolutions of the Moon	R_m	57753336
Number of revolutions of the Moon's apogee	R_{m_ap}	488219
Number of civil days	D_c	1577917500
Position of the Sun at *kalyādi*	θ_s^k	0;0,0,0[a]
Position of the Moon at *kalyādi*	θ_m^k	0;0,0,0
Position of the Moon's apogee at *kalyādi*	$\theta_{m\_ap}^k$	3;0,0,0

[a] indicates 0 *rāśis*; 0 degrees, 0 minutes, 0 seconds

Table 2: *Āryabhaṭīya* astronomical parameters for a *mahāyuga* (4320000 years)

the time difference ($t^b \sim t^d$) between the instants of true and mean sunrise at L'. Thus, applying *bhujāntara* ($\mp\left|{}^d\Delta_p^b\right|$) [or *manda* ($\mp\left|{}^d\Delta_p^m\right|$)] correction, their corresponding mean (${}^d\theta_p^b$) [or true (${}^d\theta_p^m$)] longitudes are obtained at the instant (t^b [or t^d]) of true [or mean] sunrise at L'. Subsequently, applying *manda* ($\mp\left|{}^b\Delta_p^m\right|$) [or *bhujāntara* ($\mp\left|{}^m\Delta_p^b\right|$)] correction, their corresponding true longitudes (${}^b\theta_p^m$ [or ${}^m\theta_p^b$]) are obtained at the instant (t^b) of true sunrise at L'. Up to this, the instant of (mean or true) sunrise, at L or L', is based on the assumption that the ecliptic is aligned with the celestial equator. i.e., obliquity $\epsilon = 0°$. Astronomers starting from Śrīpati (eleventh century CE) apply the *udayāntara* correction ($\mp\left|\Delta_p^u\right|$) to account for the time difference ($t^u \sim t^b$) between the instants of true sunrise at L' with and without considering the obliquity ($\epsilon = 24°$) of the ecliptic, thus obtaining the true longitudes (θ_p^u) at the instant (t^u) of the true sunrise at L'. Finally, applying the *cara* correction ($\mp\left|\Delta_p^{ca}\right|$), which accounts for the time difference ($t^{ca} \sim t^u$) between the instants of true sunrise at Q and L', gives the true longitudes of the Sun (θ_s^t) and the Moon (θ_m^t) at the instant (t^{ca}) of true sunrise at the observer's location (Q). The corrections denoted by bold arrows in Figure 1 indicate the sequence followed by *Tithinirṇaya*, whereas the dotted arrows indicate the alternate (or extra) sequence (or correction) proposed by other astronomers like Lalla, Nīlakaṇṭha Somayājin, and so on.[37] The alternate sequence is provided to compare the *Tithinirṇaya* sequence with the interpretations of Bannañje (1974*b*), and Vyāsadāsa (2007), as discussed in Section 7.1.5.

The Sections 3, 4 and 5 describe the procedure to compute the mean positions of the Sun, Moon, and Moon's apogee, respectively, at the instant (t°) of mean sunrise at Laṅkā (L). The explanations therein utilize the astronomical parameters for a *mahāyuga* prescribed by Āryabhaṭa[38] as given in Table 2.

1.6 METHODOLOGY AND CONVENTIONS

The methodology employed in translating the verses and the conventions adhered to in elucidating the content is outlined in the following discussion.

1.6.1 Translation

The discussion of the contents within *Tithinirṇaya* follows a structured approach. Initially, each relevant verse is presented in Devanagari and then transliterated into Roman script. Subsequently, an English translation is provided, with a focus on preserving the author's voice and style to the best extent possible. To enhance the correct understanding of the verses, certain words are inserted within '[ ].' Additionally, relevant phrases of the verse, such as word numerals,[39] that appear in translation, following their connotation, are enclosed in '( ).' The scribal errors and alternate readings of the verses, indicated by Bannañje (1974*b*) and Bhikṣu (n.d.), are given in the footnotes for ready reference.

1.6.2 Explanation

To help better understand the mathematical import of the verses, the mathematical expressions articulated therein are first given in Sanskrit, followed by the corresponding modern mathematical notation. The geometrical and mathematical rationales of these expressions are explained through appropriate diagrams and derivations. References to primary and secondary texts employed in our analysis are provided in the footnotes. For consistency, we have employed Sanskrit technical terms throughout the text and explained their meaning in the glossary. All the *kaṭapayādi* phrases, employed in the *Tithinirṇaya* are found to denote the attributes of Viṣṇu. The phrases, their corresponding numbers, and their meaning are summarized in Appendix B.

1.6.3 Symbols

In this work, longitudes and corrections are denoted by θ, and Δ respectively. Subscripts 's' and 'm' denote values for the Sun and the Moon, respectively, such as θ_s or θ_m for longitudes and Δ_s or Δ_m for corrections. The uncorrected mean longitudes of the Sun and the Moon are denoted by θ_s° and θ_m°, while the final corrected true longitudes are represented by θ_s^t and θ_m^t, respectively.

37 See *Śiṣyadhīvṛddhidatantra*, Chatterjee (1981: 37–39), and *Tantrasaṅgraha*, Ramasubramanian and Sriram (2011: 80–81).

38 See *Āryabhaṭīya* verses 3–4 in the *Gītikā* chapter, and verse 5 in the *Kālakriyā* chapter, Shukla and Sarma (1976: 6–7,91).

39 The numerals are encoded into words using *kaṭapayādi* system in *Tithinirṇaya*. See Ramasubramanian and Sriram (2011: 440), for more information on *kaṭapayādi* system.

Superscripts *d*, *b*, *m*, *u*, and *ca* signify corrections *deśāntara*, *bhujāntara*, *manda*, *udayāntara*, and *cara*, respectively. Post-superscripts and pre-superscripts indicate the current and previous corrections respectively. For instance, ${}^{d}\theta_s^m$ signifies the Sun's longitude resulting from a *manda* correction (${}^{d}\Delta_s^m$) performed after a *deśāntara* correction. Similarly, ${}^{b}\theta_s^m$ denotes the Sun's longitude resulting from a *manda* correction (${}^{b}\Delta_s^m$) after a *bhujāntara* correction. The notations used in this work adhere to the conventions outlined in Appendix A.

1.6.4 *Projections employed in figures*

For ease of representation, diagrams featuring geometrical entities within a sphere, such as Figures 2, 3a, 4b, 6b, 10, 18 and 19, incorporate oblique and orthographic projections. In Figure 2, for instance, the planes representing the equator and the latitudinal circle are depicted using oblique projections. Simultaneously, the Earth's axis, symbolized by the line connecting the north pole (P_N) and the south pole (P_S), is presented through an orthographic projection. This approach is consistent across all other figures.

2 INVOCATION

विष्णुं विश्वेश्वरं नत्वा तदुपोषणशुद्धये ।[40]
मूलग्रन्थानुसारेण क्रियते तिथिनिर्णयः ॥ १ ॥ ॥ अनुष्टुभ् ॥

viṣṇuṃ viśveśvaraṃ natvā taduposaṇaśuddhaye |
mūlagranthānusāreṇa kriyate tithinirṇayaḥ || 1 || || *anuṣṭubh* ||

Having venerated Viṣṇu, the lord of the universe (*viśveśvara*), for the correctness of fast [observed on *ekādaśī*, *Viṣṇupañcaka*, etc.,] for Him (Viṣṇu), [the text] *Tithinirṇaya* is composed [by me (author)] based upon the [astronomical and socio-religious] source text.

The invocation is an age-old Indian practice where the author seeks the blessings of their favorite deity (*iṣṭadevatā*) to remove the intermittent hindrances until the completion of the work. The author commences his work, titled '*Tithinirṇaya*,' with the above invocatory verse, venerating Viṣṇu, the lord of the universe. He states that the purpose of the text is to bring perfection to the practice of observing fasts, like *ekādaśī*, and *Viṣṇupañcaka*, which are performed for the sake of Viṣṇu. Further, without giving any details, the author states that this *Tithinirṇaya* is based upon an (unnamed) source text.

40 Bhikṣu (n.d.) notes an alternate reading for विश्वेश्वरम् as सर्वेश्वरम्, meaning the lord of all, including Mahālakṣmī, Brahma, etc. He also reads तदुपोषणशुद्धये as तदुपोषणसिद्धये, which in turn means the fast pertaining to Viṣṇu that leads to salvation.

2.1 EXPLANATION

2.1.1 *Purpose of the work*

The followers of Viṣṇu consider *ekādaśī* and *Viṣṇupañcaka* to be highly significant or fasting rituals (*vratas*). Śrī Madhvācārya, in his *Kṛṣṇāmṛtamahārṇava*,[41] states the importance of compulsory fast on the *ekādaśī*. Similarly, Śrī Kṛṣṇācārya, in his *Smṛtimuktāvalī*,[42] states the importance of observing the *Viṣṇupañcaka*, an optional *vrata* observed to cleanse oneself off major transgressions. Given the importance of these two *vratas*, it is pertinent that they are followed without lapse. For this, *Tithinirṇaya* lays down the rules to determine the days on which the *vratas* shall be observed. It is worth noting here that these rules are discussed only in the last 4 verses (25–28), while verses 2–24 deal with the computation of *tithi*, as it serves as a prerequisite for the application of the rules.

2.1.2 *Source text upon which the Tithinirṇaya is based*

The author, in the above verse, employs the phrase '*mūlagranthānusāreṇa*' to indicate that the *Tithinirṇaya* is based upon a source text, without providing any specifics.

Bhikṣu (n.d.) interprets *mūla-grantha* as the sole grace of Nārāyaṇa,[43] whereas Vyāsadāsa (2007: xii) interprets it as the texts that are in congruence with Vedavyāsa's thoughts.[44] Their interpretation of this phrase likely flows from their attribution of the authorship of the *Tithinirṇaya* to Śrī Madhvācārya, who, as per legend, obtained his knowledge from Vedavyāsa. However, Madhusūdana Bhikṣu also quotes verses from the texts like *Varāha-purāṇa*, *Kṛṣṇāmṛtamahārṇava*, *Sūryasiddhānta*, *Vākyakaraṇa*, and *Karaṇaprakāśa* to support his interpretation of *Tithinirṇaya*.

In our study, we observed similarities in the verses, expressions, astronomical parameters, and procedures between the *Tithinirṇaya* and earlier astronomical and religious texts. These are summarized in Table 3.

41 See Bannañje (1974*a*: 90–97).

42 See Giri Ācārya (2013: 533).

43 Another name of Lord Viṣṇu. Bhikṣu (n.d.) states अत उक्तं मूलग्रन्थेति। मूलग्रन्थस्तु श्रीनारायणकृपैव नत्वन्यः।

44 Vyāsadāsa (2007: xii) says 'ಆಚಾರ್ಯರಿಗೆ ಮೂಲಗ್ರಂಥಳೆಂದರೆ ವೇದವ್ಯಾಸದೇವರಿಗೆ ಸಮ್ಮತವಾದ ಗ್ರಂಥಗಳೇ.' Vedavyāsa, considered to be an incarnation of Viṣṇu, is a celebrated author of texts such as *Mahābhārata*, *Purāṇas*, etc., as per the Mādhva tradition. In the foreword to Vyāsadāsa (2007: III-IV), Nāgabhūṣaṇa Rao interprets *mūla-grantha* as the work which is in line with *Brahmasiddhānta*.

Verses	Text	*Similarities in*	*See footnote(s)*
2–5	*Grahacāranibandhana-saṅgraha* of Haridatta	Multipliers and divisors for computing mean longitudes and adopting revised rates of motion through *parahita* system	57, 63, 71, 64
6	*Karaṇaratna* of Devācārya	A location crossed by prime meridian	78
8–9	*Laghubhāskarīya* of Bhāskara I	The mathematical expressions of *bhujāntara* correction	92
10–11	Śaṅkaranārāyaṇa's commentary on *Laghubhāskarīya*	The verses on Rsines	119
16–18	*Karaṇaratna* of Devācārya	Verse 17, and the model to compute the motion of equinox	140, 144
25	*Tithinirṇaya*[a] of Śrī Trivikrama-paṇḍitācārya	Verse on *viddhaikādaśī*	191
26	*Bhaviṣyat-purāṇa* of Vedavyāsa	Verse on *Viṣṇupañcaka*	206
27	*Skānda-purāṇa* of Vedavyāsa	The content of the verse on reaping benefits of a fast	209
28	*Kṛṣṇāmṛtamahārṇava* of Śrī Madhvācārya	Verse on *Saṅkoca-dvādaśī*	212

[a] See Section 1.3 for our discussion on Authorship.

Table 3: Similarities between *Tithinirṇaya* and other texts

3 MEAN LONGITUDE OF THE SUN AT MEAN SUNRISE AT LAṄKĀ

भूश्रीभिन्नाकिचिन्त्योनात् कल्यहात् कालवर्धितात् ।[45]
गरुडध्येयवाक्याप्तं त्यक्त्वा सौरं वृथाफलम् ॥ २ ॥
राश्याद्यं मध्यमं कुर्याद् गोघ्नाद् धीसूनुनागजाः ।
कलास्त्यक्त्वा ध्रुवं कुर्याद् देशाधारहरार्पकम् ॥ ३ ॥[46] ॥ अनुष्टुभ् ॥

bhūśrībhinnākicintyonāt kalyahāt kālavardhitāt |
garuḍadhyeyavākyāptaṃ tyaktvā sauraṃ vṛthāphalam || 2 ||

45 Bannañje (1974*b*: 176) notes that the alternate readings such as भूगीभिन्नाकि, कल्यब्दात् and कालवर्ज्जितात् are scribal errors, which lead to wrong results. Bhikṣu (n.d.) has the reading भूश्रीभिर्नाकिचिन्त्योनात् कल्यब्दात्..।

46 Bannañje (1974*b*: 176) notes that the alternate readings such as धीसूनुनागणाः, कलाश्वत्वा and देशाधारनरार्पकम् are scribal errors. Bhikṣu (n.d.) has the reading देशाधारहरापिताः or देशाधारहयार्जित।

rāśyādyaṃ madhyamaṃ kuryād goghnād dhīsūnunāgajāḥ |
kalāstyaktvā dhruvaṃ kuryād deśādhāraharārpakam || 3 || || *anuṣṭubh* ||

Having discarded the futile result [i.e., quotient] obtained from the *kali-ahargaṇa* [that is] reduced by *bhūśrībhinnākicintya* (1610424) [and] multiplied by *kāla* (31) [and] divided by *garuḍadhyeya* (11323), may [one] do the [conversion of the fractional part into units] beginning with *rāśi*s, etc. Having subtracted [from the previous result] the minutes (*kalā*) (quotient) arising from [the *kali-ahargaṇa* that is reduced by *bhūśrībhinnākicintya* (1610424) and] multiplied by *go* (3) [and divided by] *dhīsūnunāga* (30079), apply the *dhruva* [equal to] *deśādhāraharārpakam* (11 [signs] 28 [degrees] 29 [minutes] 58 [seconds]). One may do the mean pertaining to the Sun (*madhyamam sauram*) [in this manner].

The above two verses prescribe the procedure to find the mean longitude (θ_s°) of the Sun at the instant (t°) of mean sunrise for an observer at Laṅkā (L) on the desired *kali-ahargaṇa* (A).

The following is the rule prescribed in the verses:

$$\textit{madhyamasaura} = \left[\frac{A' \times k\bar{a}la}{garu\d{d}adhyeya}\right] \text{(convert the fractional part into } r\bar{a}\acute{s}i\text{s, etc.)} - \left[\frac{A' \times go}{dh\bar{\imath}s\bar{u}nun\bar{a}ga}\right] \text{(in } kal\bar{a}\text{s)} + \left[de\acute{s}\bar{a}dh\bar{a}rahar\bar{a}rpakam\right] \text{(in } r\bar{a}\acute{s}i\text{s, etc.)},$$

or, in our notation,[47]

$$\theta_s^\circ = \left[\frac{A' \times 31}{11323}\right]_{r;d,m,s} - 0;0,\frac{A' \times 3}{30079},0 + 11;28,29,58 \qquad (1)$$

where A' corresponds to the number of mean civil days elapsed since the start of a convenient epoch, chosen in the text to be the *kali-ahargaṇa* of 1610424 (*bhūśrībhinnākicintya*). Thus,

$$A' = A - 1610424. \qquad (2)$$

It may be noted that the verses refer to the integral part of $\left[\frac{A'\times 31}{11323}\right]$ as *vṛthāphala* (futile result), perhaps because this quantity is unnecessary here. The position in *rāśi*s, degrees, minutes, and seconds is obtained from the fractional part of $\left[\frac{A'\times 31}{11323}\right]$. The *dhruva* or mean longitude (θ_s^e) of the Sun at epoch, in the same units, is stated to be 11;28,29,58, employing the *kaṭapayādi* notation *deśādhāraharārpakam*.

47 The subscript $r;d,m,s$ here indicates that the expression is to be computed in the units of *rāśi*s, degrees, minutes and seconds.

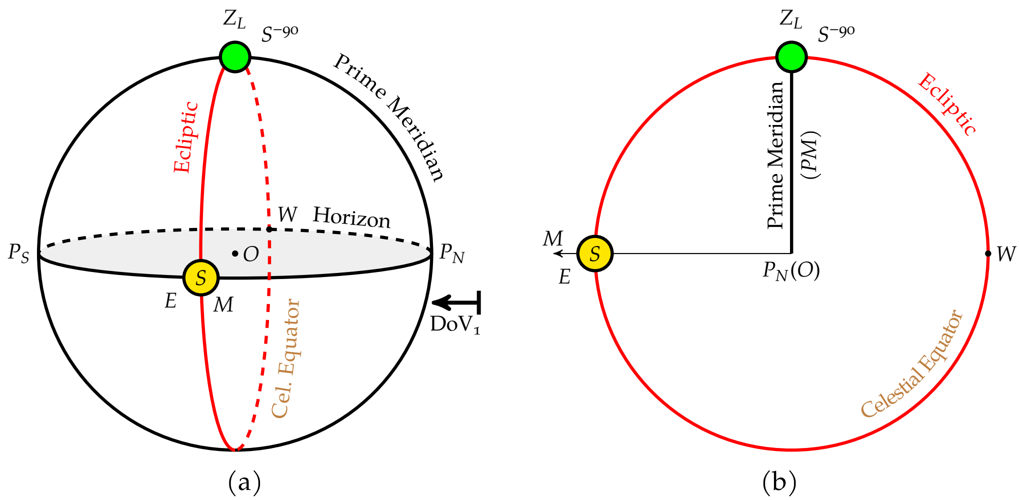

Figure 3: (a) A diagram showing a celestial sphere for an observer at Laṅkā (L) at *kalyādi*, and (b) A diagram when viewed from the Direction of View (DoV_1).

3.1 EXPLANATION

According to the *Āryabhaṭīya*, *kaliyuga* commences at mean sunrise at Laṅkā (L), and the mean Sun is located at *meṣādi* at that instant (t^k), i.e., $\theta_s^k = 0°$.[48] Figure 3 depicts the corresponding geometry of the celestial sphere for an observer at Laṅkā (L) at the instant (t^k) of *kalyādi* from two different viewpoints. Figure 3a depicts the celestial sphere from the viewpoint of the eastern horizon, in which the observer at Laṅkā (L) is located at the center (O) and his corresponding zenith is indicated as Z_L.[49] This figure further depicts an ecliptic, which is assumed to be aligned with the celestial equator, i.e., neglecting the obliquity of the ecliptic.[50] The mean Sun (S), located at *meṣādi* (M), and orbiting along the ecliptic, is just about to rise at Cardinal East (E), indicating the instant of mean sunrise. Alternatively, the mean sunrise can also be conceived to be at the instant when a fictitious body[51] S^{-90} — a point on the ecliptic which is 90° behind the Sun (S) — is on the observer's meridian.[52] Figure 3b depicts the same instant from the perspective of the northern horizon, indicated by Direction of View (DoV_1) as shown in Figure 3a.

48 See footnote 38.

49 As the radius of the Earth is considered negligible compared to the radius of the celestial sphere, the center of the Earth and the observer at Laṅkā (L) are both represented at the center (O) of the celestial sphere.

50 As the observer at Laṅkā (L) has latitude $\phi = 0$, the celestial equator is oriented perpendicular to the horizon and passes through the zenith Z_L.

51 This fictitious body S^{-90} is introduced to help explain the rationales of *deśāntara* and *bhujāntara* corrections. See Sections 6.1 and 7.1.

52 Here, it is prime meridian.

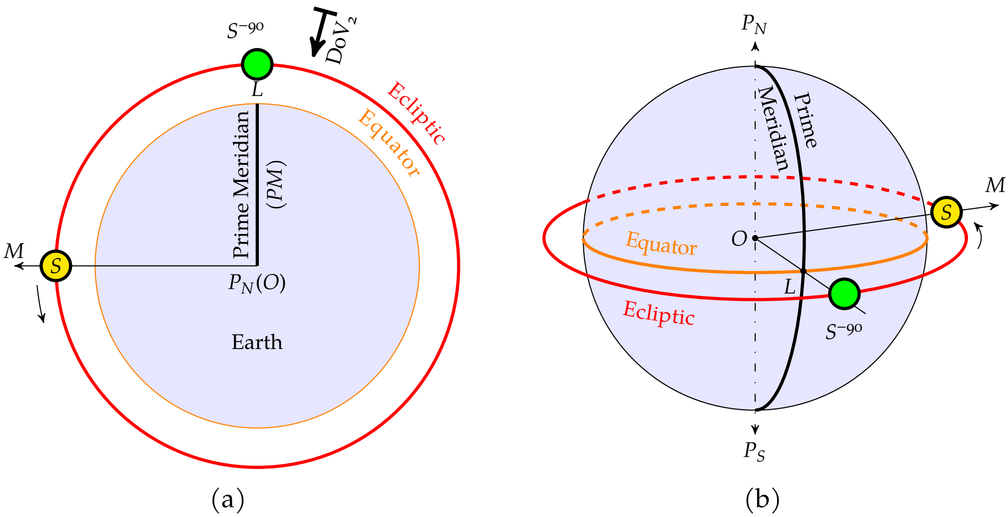

Figure 4: (a) A diagram showing the mean Sun (S) at *kalyādi*, along with its S^{-90}, orbiting around the spherical Earth along the ecliptic, and (b) A diagram when viewed from the Direction of View (DoV_2).

Alternatively, devoid of the celestial sphere in Figure 3, the same instant (t^k) of *kalyādi* and the annual motion of the mean Sun (S) around the spherical Earth is simply depicted in Figure 4.[53] Figure 4a depicts a spherical Earth[54] from the viewpoint of the north pole (P_N), showing its center (O), prime meridian (PM), and Laṅkā (L). The mean Sun (S), along with its S^{-90}, is orbiting in the ecliptic around the Earth in an anti-clockwise direction. The mean Sun (S) positioned at *meṣādi* (M) and its corresponding S^{-90} aligned with the prime meridian indicates the instant of mean sunrise at Laṅkā (L), thus depicting the instant (t^k) of *kalyādi*. Figure 4b represents the same geometry when viewed from Direction of View (DoV_2) as shown in Figure 4a. So far, the geometrical interpretation of the instant (t^k) of *kalyādi* is discussed. Now, to compute the mean longitudes at the instant (t°) of mean sunrise at Laṅkā (L) at any desired *kali-ahargaṇa* (A), consider Figure 5. This figure is similar to Figure 4a and depicts the mean longitude ($M\hat{O}S = \theta_s^\circ$) of the Sun at the same instant (t°) and its computation is explained as follows.

53 The geometric representations similar to Figure 4 are used in explanation of the corrections such as *deśāntara*, *bhujāntara*, and *manda*, hence introduced in this section. Further, the geometric equivalence of Figures 3 and 4 is utilized in the explanation of the *bhujāntara* correction.

54 The radius of the Earth is small when compared to the radius of the ecliptic hence, Earth, drawn here and in other figures, is not to the scale.

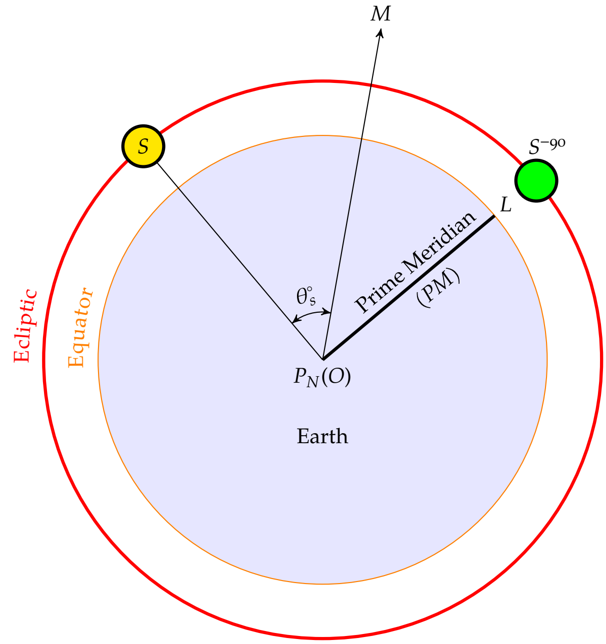

Figure 5: A diagram showing the position (θ_s°) of mean Sun at the instant (t°) of mean sunrise at Laṅkā (L) on a desired *kali-ahargaṇa* (A).

As the instant (t^k) of *kalyādi* is the mean sunrise at Laṅkā (L), and the mean civil day is the time between successive mean sunrises at the location, any desired *kali-ahargaṇa* (A) signifies the instant of mean sunrise at Laṅkā (L). Given, from the Table 2, the position (θ_s^k) of Sun at *kalyādi* to be 0°, the number of civil days (D_c) and the number of revolutions (R_s) of the Sun in a *mahāyuga* to be 1577917500 and 4320000 respectively, the mean longitude (θ_s°) of the Sun at the instant (t°) of mean sunrise for an observer at Laṅkā (L) on a desired *kali-ahargaṇa* (A) is computed as:[55]

$$\theta_s^\circ = \theta_s^k + \left[A \times \frac{R_s}{D_c}\right]_{r;d,m,s} = \left[A \times \frac{4320000}{1577917500}\right]_{r;d,m,s}, \tag{3}$$

where the ratio

$$\frac{R_s}{D_c} = \dot{\theta}_s^\circ = \frac{4320000}{1577917500}\left(\frac{\text{rev}}{\text{day}}\right) \approx 59.136\left(\frac{\text{min}}{\text{day}}\right) \tag{4}$$

55 See *Laghubhāskarīya* verses I.15–17, Shukla (1963: 5–6), and *Mahābhāskarīya* verse I.8, Shukla (1960: 6–7), *Śiṣyadhīvṛddhidatantra* verse I.17, Chatterjee (1981: 13), *Karaṇapaddhati* verse I.11, Pai, Ramasubramanian, et al. (2018: 13–14).

represents the mean rate of motion ($\dot{\theta}_s^{\circ}$) of the Sun.

The alternate approach given by the present *karaṇa* text to obtain the longitude of the mean planet is to add the position of the mean planet at the *karaṇa*'s epoch (1610424), known as *dhruva* (θ^e), to the motion of the mean planet calculated for the elapsed number of days ($A' = A - 1610424$) since epoch. For Sun, (3) can be conceived as:

$$\begin{aligned}\theta_s^{\circ} &= \theta_s^k + \left[1610424 \times \frac{R_s}{D_c}\right]_{r;d,m,s} + \left[A' \times \frac{R_s}{D_c}\right]_{r;d,m,s} \\ &= \theta_s^e + \left[A' \times \frac{R_s}{D_c}\right]_{r;d,m,s},\end{aligned} \tag{5}$$

where the *dhruva* or position (θ_s^e) of the mean Sun at the instant (t^e) of mean sunrise at Laṅkā (L) at epoch, when calculated is observed to be[56]

$$\theta_s^e = 11;28,29,57,39.85, \tag{6}$$

which is approximated to $11;28,29,58$ in the verse. The motion of the mean Sun since the epoch can be calculated as prescribed in the verse as:[57]

$$\frac{A' \times 31}{11323}\ (\text{rev}) - \frac{A' \times 3}{30079}\ (\text{min}) \approx A'\ (\text{days}) \times 59.136 \left(\frac{\text{min}}{\text{day}}\right). \tag{7}$$

As the mean rate of motion of the Sun in (7) and (4) are same and precise up to 8 decimal places,

$$\left[A' \times \frac{R_s}{D_c}\right]_{r;d,m,s} = \frac{A' \times 31}{11323}\ (\text{rev}) - \frac{A' \times 3}{30079}\ (\text{min}). \tag{8}$$

Thus, employing (6) and (8) in (5), (1) and (5) are equivalent.

56 As $\theta_s^k = 0^{\circ}$, $\theta_s^e = 1610424 \times R_s \div D_c$. The resultant quotient, 4408, represents the number of years elapsed or the number of revolutions completed by the Sun since *kalyādi* at the epoch, while the fractional part is utilized to determine the *rāśis*, and other subdivisions traversed by the Sun.

57 The similar ratio $\frac{31}{11323}$ is observed in *Grahacāranibandhana* verse I.21, and in *Grahacāranibandhanasaṅgraha* verse A.5. See Sarma (1954: 4,23).

4 MEAN LONGITUDE OF THE MOON AT MEAN SUNRISE AT LAṄKĀ

अनन्तवृद्धाद् बौधाङ्गतुल्येनेन्दुः शुकाहतात् ।[58]
प्राज्ञाञ्जलिभृदाप्तोनस्तारशोभातिनाकिनी ॥ ४ ॥[59] ॥ अनुष्टुभ् ॥

anantavṛddhād baudhāṅgatulyenenduḥ śukāhatāt |
prājñāñjalibhṛdāptonastāraśobhātinākinī || 4 || || *anuṣṭubh* ||

From the [*kali-ahargaṇa* that is reduced by 1610424 and] multiplied by *ananta* (600) [and divided] by *baudhāṅgatulya* (16393) [discard the quotient thus obtained and convert the fractional part into *rāśi*, etc.][60] [This] subtracted by [the result] obtained from [the division of] the product of [*kali-ahargaṇa* reduced by 1610424 and] *śuka* (15) by *prājñāñjalibhṛd* (43802) [and increased by the *dhruva* equal to] *tāra-śobhātinākinī* (01 [sign] 06 [degrees] 45 [minutes] 26 [seconds]) is the [mean] Moon (*indu*).

The above verse (to be read in conjunction with verses 2 and 3) prescribes the procedure to find the mean longitude (θ_m°) of the Moon at the instant (t°) of mean sunrise for an observer at Laṅkā (L) on a desired *kali-ahargaṇa* (A). The following is the rule prescribed in the verse:

$$indu = \left[\frac{A' \times ananta}{baudh\bar{a}\dot{n}gatulya}\right](\text{convert the fractional part into } r\bar{a}\acute{s}\text{is, etc.})$$
$$-\left[\frac{A' \times \acute{s}uka}{pr\bar{a}j\tilde{n}\bar{a}\tilde{n}jalibh\d{r}d}\right](\text{in } kal\bar{a}\text{s}) + [t\bar{a}ra\acute{s}obh\bar{a}tin\bar{a}kin\bar{\imath}]\,(\text{in } r\bar{a}\acute{s}\text{is, etc.}),$$

or, in our notation,

$$\theta_m^\circ = \left[\frac{A' \times 600}{16393}\right]_{r;d,m,s} - 0;0,\frac{A' \times 15}{43802},0 + 01;06,45,26, \qquad (9)$$

where A' is the elapsed number of civil days since the epoch given by (2).

58 Bannañje (1974*b*: 178) notes that the alternate readings such as अनन्तवृद्धात् त्वौधाङ्ग and तुलेनेन्दुशुकाहतात् are scribal errors. Bhikṣu (n.d.) has the reading बोधाङ्गस्तुत्येन।

59 Bannañje (1974*b*: 178) notes an alternate reading तारेशोभातिनाकनी, which does not change the result. Bhikṣu (n.d.) has the reading प्रज्ञाञ्जलिहृदाप्तोन, and suggests रोतोर्चंयाजकजनम् in place of तारशोभातिनाकिनी।

60 To be read in conjunction with verses 2 and 3.

4.1 EXPLANATION

Given, from the Table 2, the position (θ^k_m) of the Moon at *kalyādi* to be 0°, the number of civil days (D_c) and the number of revolutions (R_m) of the Moon in a *mahāyuga* to be 1577917500 and 57753336 respectively, the mean longitude (θ°_m) of the Moon at the instant (t°) of mean sunrise for an observer at Laṅkā (L) on a desired *kali-ahargaṇa* (A) is computed as:[61]

$$\theta^{\circ}_m = \theta^k_m + \left[A \times \frac{R_m}{D_c}\right]_{r;d,m,s} = \left[A \times \frac{57753336}{1577917500}\right]_{r;d,m,s}, \quad (10)$$

where the ratio

$$\frac{R_m}{D_c} = \dot{\theta}^{\circ}_m = \frac{57753336}{1577917500}\left(\frac{\text{rev}}{\text{day}}\right) \approx 790.5813\left(\frac{\text{min}}{\text{day}}\right) \quad (11)$$

represents the mean rate of motion ($\dot{\theta}^{\circ}_m$) of the Moon.

This *karaṇa* text presents (10) as:

$$\begin{aligned}\theta^{\circ}_m &= \theta^k_m + \left[1610424 \times \frac{R_m}{D_c}\right]_{r;d,m,s} + \left[A' \times \frac{R_m}{D_c}\right]_{r;d,m,s} \\ &= \theta^e_m + \left[A' \times \frac{R_m}{D_c}\right]_{r;d,m,s}, \end{aligned} \quad (12)$$

where the *dhruva* or position (θ^e_m) of the mean Moon at the instant (t^e) of mean sunrise at Laṅkā (L) at epoch, when calculated is observed to be 01;08,08,24,18.28,[62] while the value stated in the verse is 01;06,45,26. The motion of the mean Moon since the epoch can be calculated as prescribed in the verse as:[63]

$$\frac{A' \times 600}{16393}\ (\text{rev}) - \frac{A' \times 15}{43802}\ (\text{min}) \approx A'\ (\text{days}) \times 790.581\ (\text{min/day}). \quad (13)$$

The difference in the rates between (11) and (13) and in the *dhruvas* can be attributed to an additional correction called *śakābdasaṃskāra*, based upon *parahita* system, in the *Tithinirṇaya*.

61 Refer footnote 55.

62 As $\theta^k_m = 0^{\circ}$, $\theta^e_m = 1610424 \times R_m \div D_c$. The resultant quotient, 58943, represents the number of revolutions completed by the Moon since *kalyādi* at the epoch, while the fractional part is utilized to determine the *rāśis*, and other subdivisions traversed by the Moon.

63 The similarity in ratio $\frac{600}{16393}$ is observed in *Grahacāranibandhana* verse I.22, *Grahacāranibandhanasaṅgraha* verse A.8, Sarma (1954: 4,24), and in *Khaṇḍakhādyaka* verse I.10, Sengupta (1934).

The *parahita* system introduces a correction to the mean longitudes of the planets after *śaka* 444 or *kali* years 3623.[64] If S_y and K_y are the *śaka* years and *kali* years elapsed, respectively, then the correction (Δ°_{p}) to the mean longitude of a planet (p) is given by

$$\Delta^{\circ}_{p} = \left(S_y - 444\right) \times \frac{g}{h}\ (\text{min}) = \left(K_y - 3623\right) \times \frac{g}{h}\ (\text{min}), \tag{14}$$

where g and h are known as *guṇakāra* (multiplier) and *hāraka* (divisor) respectively, and takes different values for different planets. This correction is not applicable for the Sun, and therefore not employed in the computation of the mean Sun.

From (14), the annual rate and the daily rate at which the correction is applied to the mean longitude of a given planet can be inferred to be

$$\dot{\Delta}^{\circ}_{p} = \frac{g}{h}\left(\frac{\text{min}}{\text{year}}\right) = \frac{g}{h} \times \frac{4320000}{1577917500}\left(\frac{\text{min}}{\text{day}}\right). \tag{15}$$

4.1.1 *Correcting the mean rate of motion of the Moon*

The values of g and h for the Moon are stated to be 9 and 85 respectively,[65] and the correction is negatively applied to the mean rate of motion of the Moon. Thus, the corrected mean rate of motion ($\dot{\theta}^{c}_{m}$) of the Moon will be

$$\dot{\theta}^{c}_{m} = \dot{\theta}^{\circ}_{m} - \dot{\Delta}^{\circ}_{m} = 790.5813 - \frac{9}{85} \times \frac{4320000}{1577917500} \approx 790.581\left(\frac{\text{min}}{\text{day}}\right), \tag{16}$$

which is same as (13) and precise up to 8 decimal places. Hence, *Tithinirṇaya* incorporates *parahita* modified *Āryabhaṭīya* rates of motion. For a modified rate of motion $\left(\dot{\theta}^{c}_{m} = \frac{R^{c}_{m}}{D_c}\right)$ of the Moon, the modified revolutions (R^{c}_{m}) of the Moon in a *mahāyuga* will be 57753314.8. Hence,

$$\left[A' \times \frac{R^{c}_{m}}{D_c}\right]_{r;d,m,s} = \frac{A' \times 600}{16393}\ (\text{rev}) - \frac{A' \times 15}{43802}\ (\text{min}). \tag{17}$$

Thus, the revised rates are incorporated in computing the mean longitude of the Moon as:

$$\theta^{\circ}_{m} = \theta^{k}_{m} + \left[A \times \frac{R^{c}_{m}}{D_c}\right]_{r;d,m,s} = \left[A \times \frac{57753314.8}{1577917500}\right]_{r;d,m,s}. \tag{18}$$

64 See *Grahacāranibandhanasaṅgraha* verses A.17,19, Sarma (1954: 25), and *Karaṇapaddhati* verse I.12, Pai, Ramasubramanian, et al. (2018: 16–17).

65 See *Grahacāranibandhanasaṅgraha* verse A.18, Sarma (1954: 25), and *Karaṇapaddhati* verse I.12, Pai, Ramasubramanian, et al. (2018: 16–18).

4.1.2 *Correcting the dhruva of the Moon at kalyādi*

The *śakābdasaṃskāra* is incorporated to revise the rates of motion of the planets beyond the *śaka* or *kali* year 444 or 3623 respectively, but (18) also incorporates the revised rates for the days of *kaliyuga* before *kali* year 3623. As the correction incorporated for the Moon reduces its rate of motion, the reduced motion of the Moon for 3623 *kali* years will be

$$3623 \times \frac{g}{h}\ (\text{min}) = 3623 \times \frac{9}{85}\ (\text{min}), \tag{19}$$

which is added to the *dhruva* or position (θ_m^k) of the Moon at *kalyādi* to get the corrected *dhruva* (θ_m^{ck}) at *kalyādi*. Thus, the corrected position (*dhruva*) of the Moon at *kalyādi* is[66]

$$\theta_m^{ck} = \theta_m^k + 3623 \times \frac{9}{85}(\text{min}) = 0;0,0,0 + 0;0,3623 \times \frac{9}{85},0 = 0;6,23,36,42.35. \tag{20}$$

Hence, for a *parahita* corrected *Āryabhaṭīya* system, the longitude of the mean Moon is computed as:

$$\begin{aligned}\theta_m^{\circ} &= \theta_m^{ck} + \left[A \times \frac{R_m^c}{D_c}\right]_{r;d,m,s} \\ &= \theta_m^{ck} + \left[1610424 \times \frac{R_m^c}{D_c}\right]_{r;d,m,s} + \left[A' \times \frac{R_m^c}{D_c}\right]_{r;d,m,s} \\ &= \theta_m^{e} + \left[A' \times \frac{R_m^c}{D_c}\right]_{r;d,m,s}, \end{aligned} \tag{21}$$

where the *dhruva* or position (θ_m^e) of the mean Moon at the instant (t^e) of mean sunrise at Laṅkā (L) at epoch is computed to be

$$\begin{aligned}\theta_m^{e} &= \theta_m^{ck} + \left[1610424 \times \frac{R_m^c}{D_c}\right]_{r;d,m,s} \\ &= 0;6,23,36,42.35 + 1;0,21,34,12.82 \approx 01;06,45,10,55. \end{aligned} \tag{22}$$

The value given in the verse deviates from (22) by $\approx 0;0,0,16$. Hence, employing (22) and (17) in (21), (9) and (21) are equivalent.

66 See *Karaṇapaddhati* verse II.4, Pai, Ramasubramanian, et al. (2018: 57–60).

5 MEAN LONGITUDE OF THE MOON'S APOGEE AT MEAN SUNRISE AT LAṄKĀ

दिनेभ्यो द्रागरागाप्तः चन्द्रोच्चः स्यान्निभाहतात् ।[67]
जगत्सेनाङ्गलब्धोनः श्रेष्ठचिन्त्योऽम्बुनाऽर्चने ॥ ५ ॥[68] ॥ अनुष्टुभ् ॥

dinebhyo drāgarāgāptaḥ candroccaḥ syānnibhāhatāt |
jagatsenāṅgalabdhonaḥ śreṣṭhacintyo'mbunā'rcane || 5 || || *anuṣṭubh* ||

The result (remainder) obtained from [the division of the difference of *kali-ahargaṇa* and 1610424] days by *drāgarāgā* (3232), [converted into *rāśi*, etc.], subtracted by the result obtained from [the division of] the product of *nibhā* (40) [and the difference of *kali-ahargaṇa* and 1610424] days by *jagatsenāṅga* (30738) in [addition to the *dhruva* equal to] *śreṣṭhacintyo'mbunā'rcane* (06 [signs] 03 [degrees] 16 [minutes] 22 [seconds]) shall be the [mean] apogee of the Moon (*candrocca*).

The above verse (to be read in conjunction with verses 2 and 3) prescribes the procedure to find the mean longitude ($\theta^{\circ}_{m\_ap}$) of the Moon's apogee at the instant (t°) of mean sunrise for an observer at Laṅkā (L) on a desired *kali-ahargaṇa* (A). The following is the rule prescribed in the verse:

$$\begin{aligned} candrocca\d{h} = &\left[\frac{A'}{dr\bar{a}gar\bar{a}g\bar{a}}\right]\text{(convert the fractional part into } r\bar{a}\acute{s}\text{is, etc.)} \\ &- \left[\frac{A' \times nibh\bar{a}}{jagatsen\bar{a}\dot{n}ga}\right]\text{(in } kal\bar{a}\text{s)} + \left[\acute{s}re\d{s}\d{t}hacintyo'mbun\bar{a}'rcane\right]\text{(in } r\bar{a}\acute{s}\text{is, etc.),} \end{aligned}$$

or, in our notation,

$$\theta^{\circ}_{m\_ap} = \left[\frac{A'}{3232}\right]_{r;d,m,s} - 0;0,\frac{A' \times 40}{30738},0 + 06;03,16,22. \tag{23}$$

where A' is the elapsed number of civil days since the epoch given by (2).

5.1 EXPLANATION

Given, from Table 2, the position ($\theta^{k}_{m\_ap}$) of the Moon's apogee at *kalyādi* to be $3;0,0,0$, the number of civil days (D_c) and the number of revolutions (R_{m_ap}) of the Moon's apogee in a *mahāyuga* to be 1577917500 and 488219 respectively, the

67 Bannañje (1974*b*: 179) notes दिनेन as an alternate reading, and दिनेभ्यो दागरागाप्त and दिनेन घाद्रान्नरागार्तः छन्दोच्चः as scribal errors.

68 This verse is incomplete in the commentary of Bhikṣu (n.d.). Also, he proposes श्रेष्ठज्ञानोशरानये in place of श्रेष्टाचिन्त्योबुजार्चने।

mean longitude ($\theta^{\circ}_{m\_ap}$) of the Moon's apogee at the instant (t°) of mean sunrise for an observer at Laṅkā (L) on a desired *kali-ahargaṇa* (A) is computed as:[69]

$$\theta^{\circ}_{m\_ap} = \theta^{k}_{m\_ap} + \left[A \times \frac{R_{m\_ap}}{D_c}\right]_{r;d,m,s} = 3;0,0,0 + \left[A \times \frac{488219}{1577917500}\right]_{r;d,m,s}, \quad (24)$$

where the ratio

$$\frac{R_{m\_ap}}{D_c} = \dot{\theta}^{\circ}_{m\_ap} = \frac{488219}{1577917500}\left(\frac{\text{rev}}{\text{day}}\right) \approx 6.6832\left(\frac{\text{min}}{\text{day}}\right) \quad (25)$$

represents the mean rate of motion ($\dot{\theta}^{\circ}_{m\_ap}$) of the Moon's apogee.

This *karaṇa* text presents (24) as:

$$\begin{aligned}\theta^{\circ}_{m\_ap} &= \theta^{k}_{m\_ap} + \left[1610424 \times \frac{R_{m\_ap}}{D_c}\right]_{r;d,m,s} + \left[A' \times \frac{R_{m\_ap}}{D_c}\right]_{r;d,m,s} \\ &= \theta^{e}_{m\_ap} + \left[A' \times \frac{R_{m\_ap}}{D_c}\right]_{r;d,m,s}, \end{aligned} \quad (26)$$

where the *dhruva* or position ($\theta^{e}_{m\_ap}$) of the mean Moon's apogee at the instant (t^{e}) of mean sunrise at Laṅkā (L) at epoch, when calculated is observed to be 06;09,37,40,45.73,[70] while the value stated in the verse is 06;03,16,22. The motion of the mean Moon's apogee since the epoch can be calculated as prescribed in the verse as:[71]

$$\frac{A'}{3232}\,(\text{rev}) - \frac{A' \times 40}{30738}\,(\text{min}) \approx A'\,(\text{days}) \times 6.6818\,(\text{min/day}). \quad (27)$$

The difference in the rates between (25) and (27) and in the *dhruva*s are again attributed to a correction called *śakābdasaṃskāra* as explained in Section (4.1).

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69 See *Laghubhāskarīya* verses I.15–17, Shukla (1963: 5–6), and *Mahābhāskarīya* verses I.8,40, Shukla (1960: 6–7,28), *Śiṣyadhīvṛddhidatantra* verse I.17,38–39, Chatterjee (1981: 13,26), *Karaṇapaddhati* verse I.11, Pai, Ramasubramanian, et al. (2018: 13–14).

70 As $\theta^{k}_{m\_ap} = 3;0,0,0$, $\theta^{e}_{m\_ap} = 3;0,0,0 + 1610424 \times R_{m\_ap} \div D_c$. The resultant quotient, 498, represents the number of revolutions completed by the Moon's apogee since *kalyādi* at the epoch, while the fractional part is utilized to determine the *rāśi*s, and other subdivisions traversed by the Moon's apogee.

71 The similar ratio $\frac{1}{3232}$ is observed in *Grahacāranibandhana* verse I.28, *Grahacāranibandhanasaṅgraha* verse A.9, Sarma (1954: 5,24), and *Khaṇḍakhādyaka* verse I.13, Sengupta (1934).

5.1.1 Correcting the mean motion of Moon's apogee

The values of g and h for Moon's apogee are stated to be 65 and 134 respectively,[72] and the correction is negatively applied to the mean rate of motion of the Moon's apogee. Thus, the corrected mean rate of motion ($\dot{\theta}^{c}_{m\_ap}$) of the Moon's apogee will be

$$\dot{\theta}^{c}_{m\_ap} = \dot{\theta}^{\circ}_{m\_ap} - \dot{\Delta}^{\circ}_{m\_ap} = 6.6832 - \frac{65}{134} \times \frac{4320000}{1577917500} \approx 6.6818 \left(\frac{\text{min}}{\text{day}}\right), \qquad (28)$$

which is same as (27) and precise up to 8 decimal places. For a modified rate of motion $\left(\dot{\theta}^{c}_{m\_ap} = \frac{R^{c}_{m\_ap}}{D_c}\right)$ of the Moon's apogee, the modified revolutions ($R^{c}_{m\_ap}$) of the Moon's apogee in a *mahāyuga* will be 488121.9. Hence,

$$\left[A' \times \frac{R^{c}_{m\_ap}}{D_c}\right]_{r;d,m,s} = \frac{A'}{3232}\,(\text{rev}) - \frac{A' \times 40}{30738}\,(\text{min}). \qquad (29)$$

Thus, the revised rates are incorporated in computing the mean longitude of the Moon's apogee as:

$$\theta^{\circ}_{m\_ap} = \theta^{k}_{m\_ap} + \left[A \times \frac{R^{c}_{m\_ap}}{D_c}\right]_{r;d,m,s} = 3;0,0,0 + \left[A \times \frac{488121.9}{1577917500}\right]_{r;d,m,s}. \qquad (30)$$

5.1.2 Correcting the dhruva of Moon's apogee at kalyādi

Following the explanation in section 4.1.2, the corrected position (*dhruva*) of the Moon's apogee at *kalyādi* will be[73]

$$\theta^{ck}_{m\_ap} = \theta^{k}_{m\_ap} + 3623 \times \frac{65}{134}\,(\text{min}) = 3;0,0,0 + 0;29,17,25,31.34 = 3;29,17,25,31.34. \qquad (31)$$

Hence, for a *parahita* corrected *Āryabhaṭīya* system, the longitude of the mean Moon's apogee is computed as:

$$\begin{aligned}\theta^{\circ}_{m\_ap} &= \theta^{ck}_{m\_ap} + \left[A \times \frac{R^{c}_{m\_ap}}{D_c}\right]_{r;d,m,s} \\ &= \theta^{ck}_{m\_ap} + \left[1610424 \times \frac{R^{c}_{m\_ap}}{D_c}\right]_{r;d,m,s} + \left[A' \times \frac{R^{c}_{m\_ap}}{D_c}\right]_{r;d,m,s} \\ &= \theta^{e}_{m\_ap} + \left[A' \times \frac{R^{c}_{m\_ap}}{D_c}\right]_{r;d,m,s}, \end{aligned} \qquad (32)$$

72 See *Grahacāranibandhanasaṅgraha* verse A.18, Sarma (1954: 25), and *Karaṇapaddhati* verse I.12, Pai, Ramasubramanian, et al. (2018: 16–18).

73 See *Karaṇapaddhati* verse II.4, Pai, Ramasubramanian, et al. (2018: 57–60).

where the *dhruva* or position ($\theta^{e}_{m\_ap}$) of the mean Moon's apogee at the instant (t^{e}) of mean sunrise at Laṅkā (L) at epoch is computed to be

$$\theta^{e}_{m\_ap} = \theta^{ck}_{m\_ap} + \left[1610424 \times \frac{R^{c}_{m\_ap}}{D_c} \right]_{r;d,m,s}$$
$$= 3;29,17,25,31.34 + 02;03,58,59,14.5 \approx 06;03,16,24,45.85. \qquad (33)$$

The value given in the verse deviates from (33) by ≈ −0;0,0,2. Hence, employing (33) and (29) in (32), (23) and (32) are equivalent.

6 *DEŚĀNTARA* CORRECTION: TO OBTAIN MEAN LONGITUDES AT MEAN SUNRISE AT THE OBSERVER'S MERIDIAN

लङ्कास्वाम्यादिरेखायाः पूर्वपश्चिमदेशयोः ।[74]
ग्रहाणां मध्यसंस्कारलिप्ता ऋणधनं क्रमात् ॥ ६ ॥[75]
पापघ्नादध्वसङ्ख्यानादर्कलब्धविलिप्तिकाः ।[76]
अर्कस्येन्दोरनर्कघ्नात् सानुभूलब्धलिप्तिकाः ॥ ७ ॥[77] ॥ अनुष्टुभ् ॥

laṅkāsvāmyādirekhāyāḥ pūrvapaścimadeśayoḥ |
grahāṇāṃ madhyasaṃskāraliptā ṛṇadhanaṃ kramāt || 6 ||
pāpaghnādadhvasaṅkhyānādarkalabdhaviliptikāḥ |
arkasyendoranarkaghnāt sānubhūlabdhaliptikāḥ || 7 || *|| anuṣṭubh ||*

In the regions to the east and west of the meridian [passing through] Laṅkā, Svāmīnagara (*svāmya*),[78] etc., the *liptis* [obtained] from the mean [*deśāntara*] correction of the planets [shall be] negative and positive respectively. The *viliptis* obtained from the division of the product of the magnitude of longitudinal separation (*adhva*) [in *yojanas*] and *pāpa* (11) by *arka* (10) [shall be] of the Sun. The *liptis* obtained from the division of the product of the longitudinal separation in *yojanas* and *anarka* (100) by *sānubhū* (407) [shall be] of the Moon.

74 Bannañje (1974*b*: 181) states लङ्कास्वाम्यादिरेखायाः to be incomprehensible and suggests लङ्कावन्त्यादिरेखायाः as the possible reading.

75 Bannañje (1974*b*: 181) notes ऋणधनक्रमात् as an alternate reading and गणानां मध्यसंस्कार as a scribal error. Bhikṣu (n.d.) has the reading मध्यसंस्कारो लिप्तातृणधनम्।

76 Bhikṣu (n.d.) has the reading चापघ्नात्।

77 This half of the verse is missing in Bhikṣu (n.d.).

78 See *Karaṇaratna* verse I.30, Shukla (1979: 21–22), where Shukla recognizes the modern Svāmīhalli (14.97°N, 76.57°E) is located in the Hospet district of Karnataka. Vyāsadāsa (2007: 13–15) refers *svāmya* to be dominion. Hence, Laṅkā-*svāmya* indicates the dominion of Laṅkā, where (*ādi*) etc., refers to other places like Avantī, and so on. Bannañje (1974*b*: 181) prefers the reading Avantī in place of *svāmya*.

The above two verses prescribe the *deśāntara* corrections (Δ_s^d and Δ_m^d) for the Sun and the Moon. This correction accounts for the time difference ($\Delta t^d = t^d \sim t^\circ$) between the instants of mean sunrise at the observer's meridian (or at L') and prime meridian (or at L). The following are the two rules prescribed in the verses:

$$\Delta_s^d = \frac{adhva \times p\bar{a}pa}{arka}\ (vilipti) = \frac{\Delta d \times 11}{10}\ (\text{sec}) \tag{34}$$

$$\Delta_m^d = \frac{adhva \times anarka}{s\bar{a}nubh\bar{u}}\ (lipti) = \frac{\Delta d \times 100}{407}\ (\text{min}), \tag{35}$$

where *adhva* (Δd) refers to the distance in *yojanas* between the longitudes of the prime meridian and the observer's meridian along the equator.

The mean longitudes (θ_s^d and θ_m^d) of the Sun and the Moon, at the instant (t^d) of mean sunrise for an observer on the equator (L'), Δd *yojanas* (east or west) from Laṅkā (L) as shown in Figure 2, is stated to be:

$$\theta_s^d = \theta_s^\circ \mp \left|\Delta_s^d\right| \tag{36}$$

$$\theta_m^d = \theta_m^\circ \mp \left|\Delta_m^d\right|, \tag{37}$$

where θ_s° and θ_m° are the mean longitudes obtained from (1) and (9) respectively. The corrections are to be subtracted for locations east of the prime meridian and added for locations to the west.

6.1 EXPLANATION

Sections 3, 4, and 5 result in the mean longitudes (θ_s°, θ_m°, and $\theta_{m\_ap}^\circ$) of the Sun, Moon, and Moon's apogee, respectively, at the instant (t°) of mean sunrise at Laṅkā (L). Now, their corresponding mean longitudes at the instant (t^d) of mean sunrise at L', which is either east or west of Laṅkā (L) by Δd *yojanas* as shown in Figure 2, have to be determined. The *deśāntara* correction aims to compute the time difference (Δt^d) in the instants (t° and t^d) of mean sunrise between the prime meridian and the observer's meridian, and further obtain the mean longitude of the planet (p)[79] at the instant (t^d) of mean sunrise at the observer's meridian (or at L').

The rationale for this correction can be understood with the help of Figure 6, which is similar to Figure 4 and depicts the diurnal motion of the mean Sun (S).[80] Figure 6a depicts a spherical Earth from the viewpoint of the north pole

79 Here, planet (p) could be replaced with Sun (s), Moon (m) or Moon's apogee (m_ap).

80 The direction of the orbital motion of the mean Sun (S) is always eastwards with respect to the observer at Laṅkā (L), anti-clockwise in Figure 4, whereas the diurnal motion of the mean Sun (S), which happens because of the rotation of the Earth, is always westwards, clockwise in Figure 6.

(P_N), showing the prime meridian (PM) and Laṅkā (L). It further depicts two meridians, which are east and west of the prime meridian (PM), by Δl degrees or Δd *yojana*s along the equator.[81] The diurnal motion of the mean Sun, which is always westwards, is indicated by the positions of the mean Sun S_1, S_2 and S_3 at the time instants t_1, t_2 and t_3 respectively, where $t_1 < t_2 < t_3$. The corresponding fictitious bodies S_1^{-90}, S_2^{-90} and S_3^{-90} are also depicted in the figure. Figure 6b presents the same geometry when viewed from the Direction of View (DoV) shown in Figure 6a.

The time instants t_1, $t_2 = t^\circ$, and t_3 indicate the instants of mean sunrise for the meridians east of PM, prime meridian, and west of PM, respectively. Thus, the corresponding fictitious bodies S_1^{-90}, S_2^{-90} and S_3^{-90} are aligned with the meridians east of PM, prime meridian, and west of PM, respectively.

If $\Delta t^d = t^d \sim t^\circ$ is the time difference between the instants of mean sunrise at the observer's meridian and the prime meridian, the angle traversed by the planet for the period of Δt^d is known as *deśāntara* correction (Δ_p^d). As the sunrise occurs earlier or later at the meridians, which are to the east or west of the PM, respectively, the correction must be accordingly subtracted or added. Hence, the mean longitude (θ_p^d) of the planet corrected for *deśāntara* is given by

$$\theta_p^d = \theta_p^\circ \mp \left|\Delta_p^d\right|, \tag{38}$$

which is equivalent to (36) and (37), the formulae prescribed in the verse, when the Sun (s) and Moon (m) are substituted for the planet (p) respectively.

In a mean civil day,[82] the mean Sun takes 3600 *vighaṭikā*s to cover 360° corresponding to the circumference (C) of the Earth (successive transits of the meridian). The time, in *vighaṭikā*s, taken by the Sun to transit between two meridians apart by Δl degrees or Δd *yojana*s along the equator will be

$$\Delta t^d = \frac{\Delta l \times 3600}{360^\circ}\ (\textit{vighaṭikās}) = \frac{\Delta d \times 3600}{C}\ (\textit{vighaṭikās}). \tag{39}$$

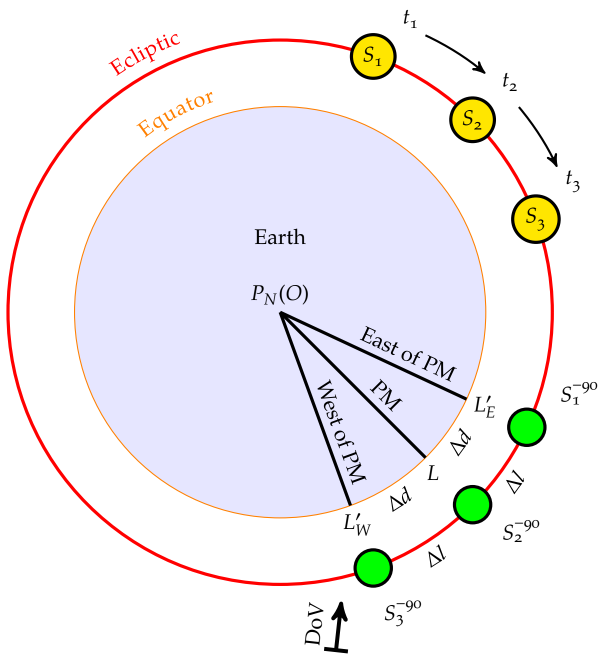
Thus, the *deśāntara* correction for the planet (Δ_p^d) — the angle traversed by the planet during the period Δt^d *vighaṭikā*s — will be[83]

$$\Delta_p^d = \frac{\Delta t^d \times \dot{\theta}_p^\circ}{3600}\ (\text{min}) = \frac{\Delta d \times \dot{\theta}_p^\circ}{C}\ (\text{min}), \tag{40}$$

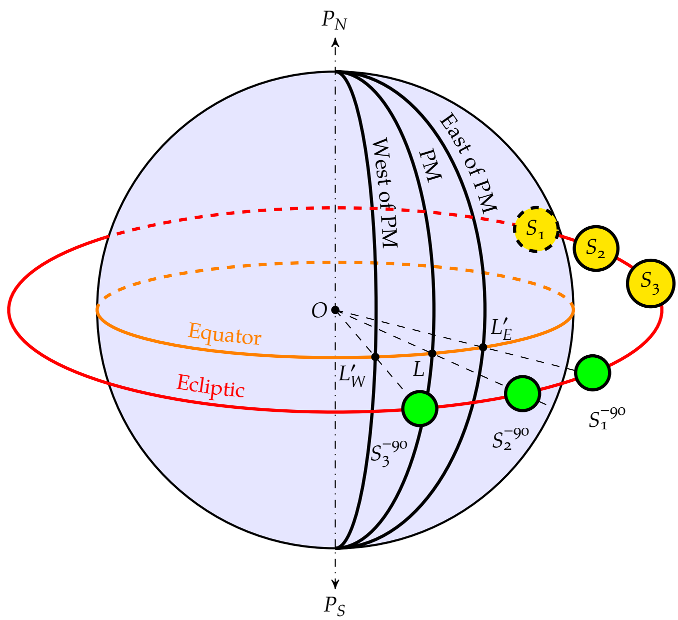
81 If L'_E and L'_W are the points of intersection of the equator and the meridians east and west of prime meridian (PM) respectively, then $LL'_E = LL'_W = \Delta d$ *yojana*s.

82 1 mean civil day = time between two successive mean sunrises = 60 *ghaṭikā*s (*nāḍikā*s) = 3600 *vighaṭikā*s (*vināḍikā*s).

83 See *Laghubhāskarīya* verses I.31–33, Shukla (1963: 11–12), *Mahābhāskarīya* verse II.10, Shukla (1960: 55), *Khaṇḍakhādyaka* verse I.15, Sengupta (1934), *Karaṇaratna* verse I.27, Shukla (1979: 20), *Laghumānasa* verse IV.3, Shukla (1990: 140), *Śiṣyadhīvṛddhidatantra* verses I.44–45, Chatterjee (1981: 31). Also see *Tantrasaṅgraha* section I.14, Ramasubramanian and Sriram (2011: 40–43).



(a)



(b)

Figure 6: (a) A diagram showing the diurnal motion of the mean Sun, indicating the positions of the mean Sun S_1, S_2 and S_3 at the instants of mean sunrise at meridians east of prime meridian (PM), PM and west of PM respectively and (b) A diagram when viewed from the Direction of View (DoV).

where $\dot{\theta}_p^\circ$ is the mean rate of motion of the planet in min/day.

As evident from (40), the *deśāntara* correction requires the knowledge of the circumference of the Earth (C), which is not stated in *Tithinirṇaya*. However, it can be inferred by comparing the *deśāntara* correction given in the verse for the Moon with the derived expression, i.e., comparing (40) and (35) and employing (16), we have

$$\frac{\Delta d \times \dot{\theta}_m^c}{C} = \frac{\Delta d \times 100}{407} \implies C \approx 3217.6 \text{ (\textit{yojanas})}. \tag{41}$$

Further, considering the *deśāntara* correction for the Sun, comparing (40) and (34) and employing (41), we have

$$\frac{\Delta d \times \dot{\theta}_s^\circ}{C} = \frac{\Delta d \times 11}{10 \times 60} \implies \dot{\theta}_s^\circ \approx 58.99 \text{ (min/day)}, \tag{42}$$

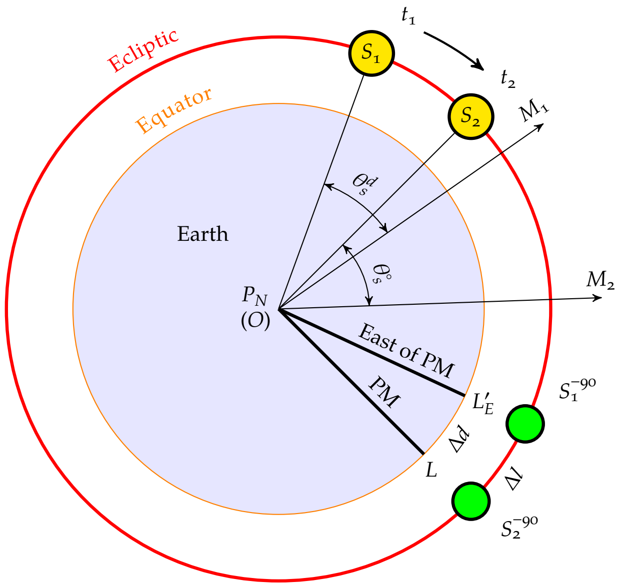
which is approximately equal to θ_s° in (4).[84]

Bannañje (1974*b*: 182) and Vyāsadāsa (2007: 15) consider the Earth's circumference (C) to be 3300 *yojanas*,[85] which results in the mean rates of motion ($\dot{\theta}_s^\circ$ and $\dot{\theta}_m^c$) of the Sun and the Moon to be approximately 60.5 (min/day) and 810.8 (min/day) respectively. Recognizing that these values are significantly different from (4) and (16), Bannañje (1974*b*: 182) suggests that the divisor in (35) be taken as 417, by reading *sānubhū* as *sānyabhū* in verse 7, which would yield the Moon's mean rate of motion to be ≈ 791.36 (min/day), which is closer to the rate in (16). However, Bannañje (1974*b*: 182) does not address the error in the Sun's mean rate of motion when considering the Earth's circumference to be 3300 *yojanas*. Thus, from the analysis, considering the circumference of the Earth (C) to be ≈ 3218 *yojanas* reduces the error of computing the *deśāntara* correction significantly.

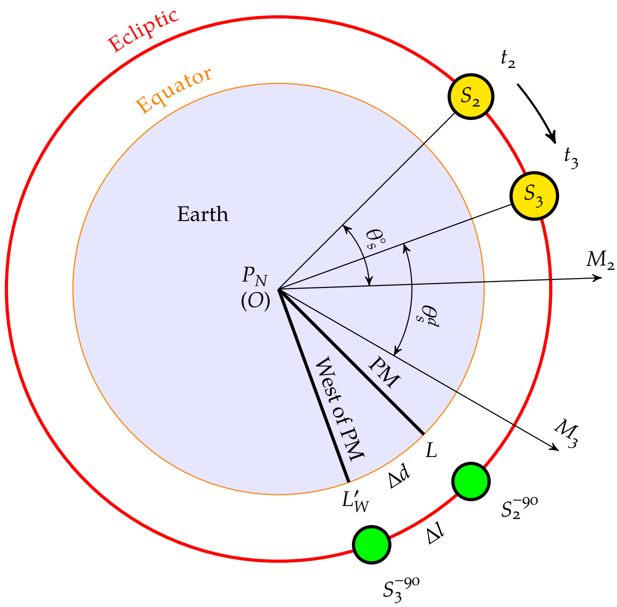
It is worth noting that this text has not addressed the *deśāntara* correction for the Moon's apogee. The *deśāntara* correction ($\Delta^d_{m\_ap}$) for the Moon's apogee is obtained by substituting (28), the mean rate of motion ($\dot{\theta}^c_{m\_ap}$) of the Moon's apogee, in (40).

84 Alternatively, first comparing the *deśāntara* correction for the Sun, i.e., comparing (40) and (34) and employing (4), the Earth's circumference (C) is computed to be 3225.6 *yojanas*. Further comparing the *deśāntara* correction for the Moon, i.e., comparing (40) and (35) and employing $C = 3225.6$ *yojanas*, we obtain $\dot{\theta}_m^c = 792.53$ (min/day), which is off from (16) by ≈ 2(min/day).

85 See *Laghubhāskarīya* verse I.24, Shukla (1963: 8), *Śiṣyadhīvṛddhidatantra* verse I.43, Chatterjee (1981: 29), and *Tantrasaṅgraha* verse I.29, Ramasubramanian and Sriram (2011: 40–41). Also see Shukla (1960: 50–51).



(a)



(b)

Figure 7: The diagrams depicting the mean longitudes of the Sun, θ_s° and θ_s^d, before and after *deśāntara* correction respectively for the meridians (a) east of PM and (b) west of PM.

As the motion of the Moon's apogee, given by (28), is relatively slow ($\dot{\theta}^{c}_{m\_ap} \approx$ 6.681 min/day), its corresponding *deśāntara* correction ($\Delta^{d}_{m\_ap}$) is small, and thus probably neglected by the author.

The geometrical implication of the *deśāntara* correction for the Sun is depicted in Figure 7. This figure depicts the *deśāntara* corrected Sun (θ^{d}_{s}) at mean sunrise at the observer's meridian, which is either east or west of the prime meridian by Δl degrees or Δd *yojana*s. Figures 7a and 7b are similar to Figure 6a and depict the instants of mean sunrise for meridians east and west of the prime meridian, respectively. If M_1, M_2, and M_3 are the successive positions of *meṣādi*, during the diurnal motion, at time instants t_1, t_2, and t_3 respectively, and $M_2\hat{O}S_2 = \theta^{\circ}_{s}$ is the mean longitude of the Sun at the instant (t°) of mean sunrise at Laṅkā (L), then the mean longitudes of the Sun (θ^{d}_{s}) at the instant ($t^{d} = t_1$ and t_3) of mean sunrise at L'_E and L'_W are indicated by $M_1\hat{O}S_1$ and $M_3\hat{O}S_3$ in Figures 7a and 7b respectively.

7 SUN'S APOGEE AND *BHUJĀNTARA* CORRECTION: TO OBTAIN MEAN LONGITUDES AT TRUE SUNRISE AT THE OBSERVER'S MERIDIAN

अर्कदोर्ज्याफलाच्छुद्धात् यथाऽर्के तत्फलात् तथा ।[86]
गोघ्नाद् दिव्यप्रजाभ्यां विलिप्ता लिप्ता इनाब्जयोः ॥ ८ ॥[87]
देशदोःशुद्धये दानक्षिष्णुभ्यां स्युर्विलिप्तिकाः ।[88]
उच्चं सूर्यस्य नियतं दुष्टास्त्री-भागराशयः ॥ ९ ॥ [89] ॥ अनुष्टुभ् ॥

arkadorjyāphalācchuddhāt yathā'rke tatphalāt tathā |
goghnād divyaprajābhyāṃ viliptā liptā inābjayoḥ || 8 ||
deśadoḥśuddhaye dānakṣiṣṇubhyāṃ syurviliptikāḥ |
uccaṃ sūryasya niyataṃ duṣṭāstrī-bhāgarāśayaḥ || 9 || || *anuṣṭubh* ||

As in the case of the [*manda* corrected] Sun [obtained] from correcting [the *deśāntara* corrected Sun] for the Sun's equation of center (*arkadorjyāphala*), similarly, from that result (Sun's equation of center) multiplied by *go* (3) and divided by *divya* (18) and *praja* (82)

86 Bhikṣu (n.d.) has the reading अर्कदोर्जाफलालब्धं यथार्के तत्फलम्..।

87 Bannañje (1974*b*: 182) notes that the alternate readings दिव्यप्रजाभ्यां लिप्तिकालिप्त इनाब्जयोः and विलिप्ता लिप्त इनाब्जयोः are scribal errors. Bhikṣu (n.d.) has the reading गोघ्नं दिव्यप्रजानाभ्याम्..।

88 Bannañje (1974*b*: 183) notes the alternate readings: देशयोः शुद्धये, देशदोः शुचये, and दानक्षिष्णुभ्याम् but states the verse 9(a,b) to be unclear. Instead of दानक्षिष्णुभ्याम्, he proposes दानक्ष्विष्णुभ्याम्। Vyāsadāsa (2007: 13) proposes दानक्षिष्णुभ्याम् and discusses its etymology. Bhikṣu (n.d.) has the reading देशयोः शुद्धये दानविष्णुभ्याम्..।

89 Bannañje (1974*b*: 183) notes दृष्टा स्त्री as an alternate reading and भागनाशयोः as a scribal error. Bhikṣu (n.d.) has the reading दृष्टे श्री भागे राशयः।

> [the *bhujāntara* correction] in seconds (*viliptis*) and minutes (*liptis*) of the Sun and the Moon [respectively] are obtained by addition and subtraction [to the *deśāntara* corrected Sun and Moon] for the correction of true sunrise at the location.[90] The Sun's apogee (*sūryasya uccaṃ*) is always *duṣṭāstrī* (2–18) signs-degrees (i.e., 2; 18, 0, 0).

Verse 9(c,d) states that the longitude of the Sun's apogee (θ_{s_ap}) is assumed constant and given to be[91]

$$\theta_{s\_ap} = \textit{duṣṭāstrī} = 2;18,0,0 = 78^\circ. \tag{43}$$

The verses 8, 9(a,b) prescribe the *bhujāntara* corrections (${}^{d}\Delta_s^b$ and ${}^{d}\Delta_m^b$) for the *deśāntara* corrected mean Sun (θ_s^d) and mean Moon (θ_m^d). This correction accounts for the time difference ($\Delta t^b = t^b \sim t^d$) between the instants of the true and mean sunrise at L', and results in the mean Sun (${}^{d}\theta_s^b$) and the mean Moon (${}^{d}\theta_m^b$) at the instant (t^b) of true sunrise at L'. To this end, the following rules are prescribed in the above verses:[92]

$$\begin{aligned} {}^{d}\theta_s^b = \theta_s^d \pm \left|{}^{d}\Delta_s^b\right| &= \theta_s^d \pm \left|\textit{arkadorjyāphala} \times \frac{90}{\textit{divya}}\right| \text{(in \textit{vilipti})} \\ &= \theta_s^d \pm \left|{}^{d}\Delta_s^m \times \frac{3}{18}\right| \text{(in sec)} \end{aligned} \tag{44}$$

$$\begin{aligned} {}^{d}\theta_m^b = \theta_m^d \pm \left|{}^{d}\Delta_m^b\right| &= \theta_m^d \pm \left|\textit{arkadorjyāphala} \times \frac{90}{\textit{praja}}\right| \text{(in \textit{lipti})} \\ &= \theta_m^d \pm \left|{}^{d}\Delta_s^m \times \frac{3}{82}\right| \text{(in min)}, \end{aligned} \tag{45}$$

where θ_s^d and θ_m^d are the values obtained from (36) and (37) respectively and ${}^{d}\Delta_s^m$ is the *deśāntara* corrected Sun's equation of center. The verses further state that the sign of the above corrections is the same as the sign employed in the *manda* correction of the Sun.

90 The additional word *viliptikāḥ* seems to be redundant in the verse.

91 See *Āryabhaṭīya* verse 9 in the *Gītikā* chapter, Shukla and Sarma (1976: 19), *Laghubhāskarīya* verse I.22, Shukla (1963: 7), *Mahābhāskarīya* verses VII.11–12, Shukla (1960: 206), *Karaṇaratna* verse I.10, Shukla (1979: 6), *Śiṣyadhīvṛddhidatantra* verse II.9, Chatterjee (1981: 35), and *Tantrasaṅgraha* verse I.40, Ramasubramanian and Sriram (2011: 46).

92 The same expressions of the correction term for the Sun and Moon could be observed in *Laghubhāskarīya* verse II.5, Shukla (1963: 19).

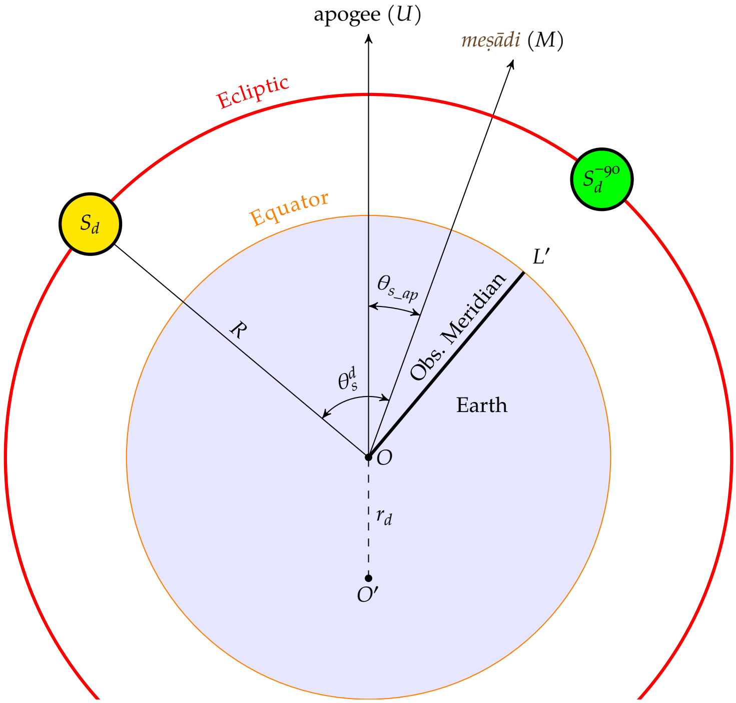


Figure 8: A diagram showing the *deśāntara* corrected Sun (S_d), its apogee (U), and its S_d^{-90} at the instant (t^d) of mean sunrise at L'.

7.1 EXPLANATION

Up to this point, from (36), (37), (43) and (23), we have computed the *deśāntara* corrected mean longitudes of the Sun (θ_s^d) and the Moon (θ_m^d) and their respective apogees $(\theta_{s\_ap}$ and $\theta^{\circ}_{m\_ap})$ at the instant (t^d) of mean sunrise at L'. All these longitudes are measured with respect to an observer positioned at the center of their respective orbits.[93] In fact, the prefix 'mean' to any parameter indicates its measure with respect to the observer positioned at the center of the orbit. However, as the apogee is the farthest point in the orbit, the true observer cannot be located at the center, but at a point further along the line joining the apogee and the center. For example, for the Sun, the above geometry can be understood with the help of Figure 8. This figure is similar to Figure 7 and depicts the *deśāntara* corrected Sun (S_d) and its apogee (U) orbiting in the ecliptic. Their mean longitudes, measured with respect to the observer at the center (O) of the ecliptic, are

93 The orbits of the apogees of the Sun and the Moon are considered to be the orbits of the Sun and the Moon themselves respectively.

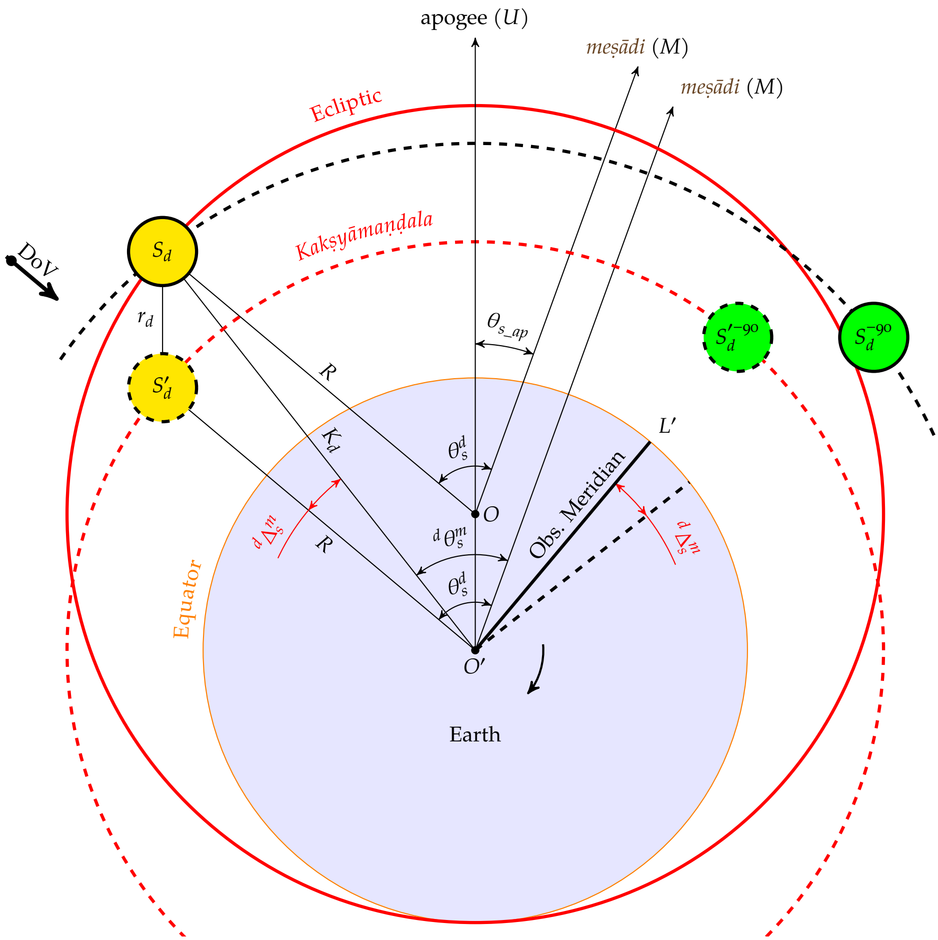


Figure 9: A diagram showing the effect of the shift in the observer from the mean position (O) to the true position (O') at the instant (t^d) of mean sunrise at L'.

given by $M\hat{O}S_d = \theta^d_s$ and $M\hat{O}U = \theta_{s\_ap}$ respectively. Further, the observer's meridian is aligned with S^{-90}_d indicating the instant of mean sunrise at L'. Now, the true observer is located at O', at a distance of $OO' = r_d$ in the direction opposite to the apogee (U) from the center of the orbit.

The effect of the true observer being positioned at O' is explained with the help of Figure 9, which depicts the Earth to be now centered at O'. At the instant of mean sunrise at L', the true observer at O' views the *deśāntara* corrected Sun (S_d) at an angle $M\hat{O}'S_d = {}^d\theta^m_s$, which is the true longitude of the Sun (S_d), situated at a distance $O'S_d = K_d$, known as *manda-karṇa*. The true observer at O' also views the observer's meridian not aligned with S^{-90}_d, indicating that this is not the

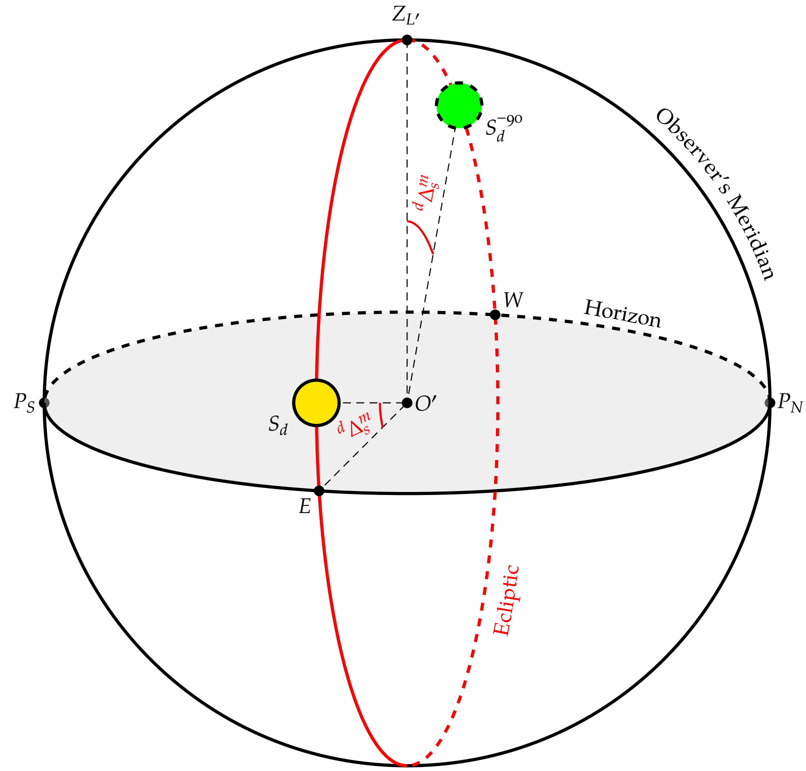


Figure 10: A diagram showing the apparent shift in the position of the Sun (S_d) from the horizon due to shift in the position of the observer from O to O'.

instant of sunrise. As we are interested in the instant of true sunrise, where the prefix 'true' indicates its measure with respect to the true observer (O'), the time difference (Δt^b) between the instant of mean and true sunrise has to be determined. To compute this difference, consider two fictitious bodies S_d' and $S_d'^{-90}$ in an orbit (*kakṣyāmaṇḍala*) centered at O', having the same radius ($O'S_d' = OS_d = R$) as the ecliptic. Let S_d' have the same longitude as the *deśāntara* corrected Sun, i.e., $M\hat{O}'S_d' = \theta_s^d$, which implies that its $S_d'^{-90}$ is aligned with the meridian of the true observer. From the geometry, the true longitude ($^d\theta_s^m$) of the *deśāntara* corrected Sun (S_d) at the instant (t^d) of mean sunrise at L' is computed to be

$$\begin{aligned} {}^d\theta_s^m = M\hat{O}'S_d = M\hat{O}'S_d' - S_d\hat{O}'S_d' \\ = \theta_s^d - {}^d\Delta_s^m, \end{aligned} \tag{46}$$

where $^d\Delta_s^m$ is the equation of center for the *deśāntara* corrected Sun (S_d).[94]

94 See (61) for the expression of the equation of center.

Further, the misalignment of S_d^{-90} with the observer's meridian, due to the observer's shift from O to O', indicated by $S_d'^{-90}\hat{O}'S_d^{-90}$, is also observed to be ${}^d\Delta_s^m$.[95] This misalignment of S_d^{-90} with the observer's meridian can be better perceived with the help of Figure 10. This figure depicts a celestial sphere with the true observer (O') at its center, having the same geometry as Figure 9 when viewed from the indicated Direction of View (DoV).[96] At the instant (t^d) of mean sunrise at L', the position of S_d^{-90} is observed to be away from the observer's meridian by $Z_{L'}\hat{O}'S_d^{-90} = {}^d\Delta_s^m$, which implies that the Sun (S_d) is displaced from the horizon by $E\hat{O}'S_d = {}^d\Delta_s^m$, thus indicating the Sun (S_d) has already risen. As we are interested in the instant (t^b) of true sunrise, one should travel back in time to observe the Sun at the horizon. This can be approximately[97] achieved by fixing the position of the Sun (S_d) and rotating[98] the Earth by ${}^d\Delta_s^m$ (clockwise in Figure 9) such that the observer's meridian aligns with S_d^{-90}.[99] If Δt^b is the sidereal time taken for the rotation of the Earth by ${}^d\Delta_s^m$, which is the time difference ($t^b \sim t^d$) between the instants of true and mean sunrise at L', the angle traversed by the planet in this time interval is known as the *bhujāntara* correction (Δ_p^b) of the planet. As the Earth rotates 360° or 21600′ in a sidereal day, the time taken by the Earth to rotate by ${}^d\Delta_s^m$ (min) will be

$$\Delta t^b = \frac{{}^d\Delta_s^m}{21600'} \text{ (sidereal day).} \tag{47}$$

Thus, the *bhujāntara* correction (Δ_p^b) of the planet — the angle (Δ_p^b), in minutes, traversed by the planet in the time interval Δt^b — will be[100]

$$\Delta_p^b = \Delta t^b \times \dot{\theta}_p = \frac{{}^d\Delta_s^m}{21600'} \times \dot{\theta}_p \text{ (min),} \tag{48}$$

where $\dot{\theta}_p$ is the rate of motion of the planet in min/day.[101] In what follows, we discuss the application of (48) in obtaining the true longitudes of planets at true sunrise at L'.

There can be two possible approaches, where the *bhujāntara* correction could be applied before or after the *manda* correction. In this work, we employ the notations ${}^b\theta_s^m$ and ${}^m\theta_s^b$ to denote the true longitude of the Sun at the instant of

95 As the angle between perpendiculars are equal, $S_d'^{-90}\hat{O}'S_d^{-90} = S_d\hat{O}'S_d' = {}^d\Delta_s^m$.

96 Like Figure 3a presents the same geometry as Figure 4a.

97 See Section 7.1.4 for our discussion on approximation.

98 To go back in time to the instant of sunrise, we need to adjust for the diurnal motion of the true Sun. Here, we achieve it by rotating the Earth instead.

99 The alignment of observer's meridian with S_d^{-90} after the sidereal rotation of the Earth by ${}^d\Delta_s^m$ is shown in Figure 11.

100 See *Laghubhāskarīya* verses II.4, 22, Shukla (1963: 18–19,28), *Mahābhāskarīya* verses IV.7, 24, 29–30, Shukla (1960: 114, 126–127,129), *Śiṣyadhīvṛddhidatantra* verse II.16, Chatterjee (1981: 37–38).

101 Strictly speaking, the units of the rate of motion should be in min/sidereal day.

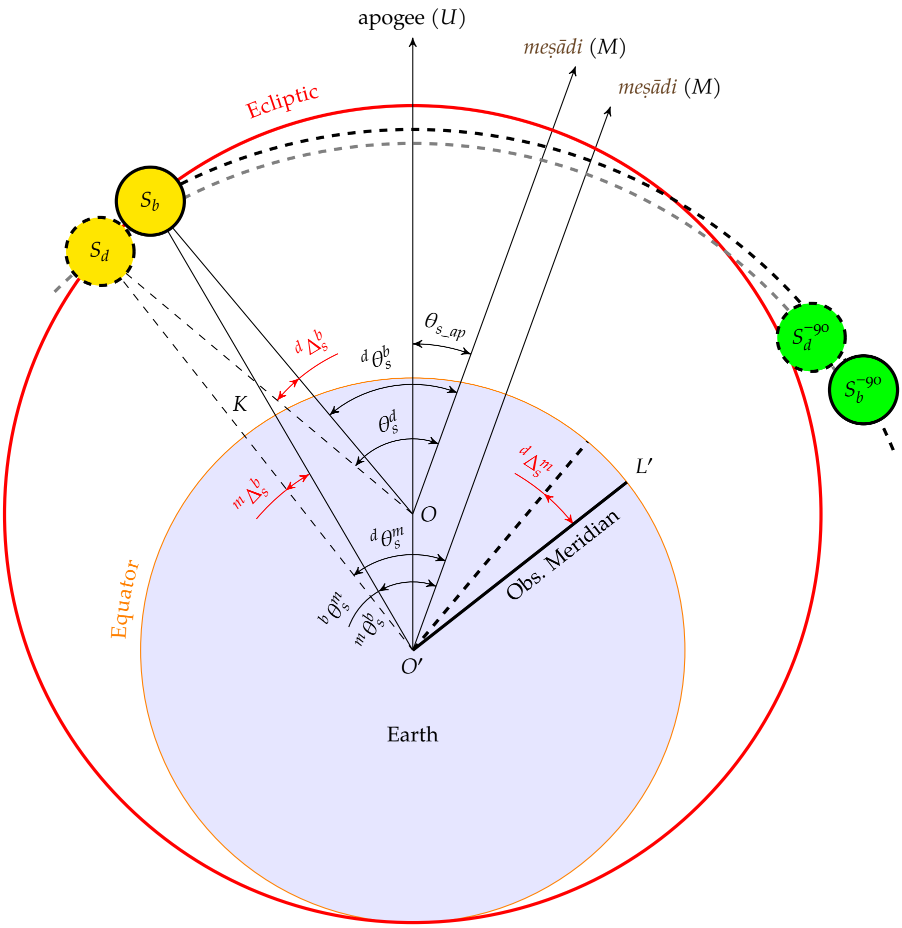


Figure 11: A diagram showing the motion of the Sun from S_d to S_b during the sidereal rotation of the Earth by ${}^{d}\Delta_s^m$.

true sunrise at L',[102] obtained from the former and later approaches respectively. Both these approaches, in the case of the Sun, can be understood with the help of Figure 11. This figure, which is similar to Figure 9, depicts a spherical Earth, which has been rotated by ${}^{d}\Delta_s^m$ to align the observer's meridian with S_d^{-90}. During the time of rotation (Δt^b), the Sun has moved in its orbit to a new position S_b indicated by $M\hat{O}S_b = {}^{d}\theta_s^b$ and $M\hat{O}'S_b = {}^{m}\theta_s^b$, with respect to observers at O and O' respectively. The two quantities ${}^{d}\theta_s^b$ and ${}^{m}\theta_s^b$ represent the mean and true longitudes of the *bhujāntara* corrected Sun (S_b) at true sunrise at L'. Section 7.1.1

102 Note that ${}^{b}\theta_s^m = {}^{m}\theta_s^b$. See Section 1.6.3 for understanding the conventions used for symbols.

discusses the computation of the true longitude (${}^{m}\theta_s^b$) of the Sun by applying the *bhujāntara* correction to the *manda* corrected true Sun (${}^{d}\theta_s^m$). Section 7.1.2 discusses the computation of the true longitude (${}^{b}\theta_s^m$) of the Sun by applying the *manda* correction to the *bhujāntara* corrected mean Sun (${}^{d}\theta_s^b$), which is the method found in *Tithinirṇaya*.

7.1.1 *The bhujāntara correction applied after manda correction*

Having known from (46) the true longitude (${}^{d}\theta_s^m$) of the *deśāntara* corrected Sun (S_d) at the instant (t^d) of mean sunrise at L', indicated by $M\hat{O}'S_d$ in Figure 11, the true longitude (${}^{m}\theta_s^b$) of the Sun (S_b) at the instant (t^b) of true sunrise at L' is obtained by applying the *bhujāntara* correction in the following manner:[103]

$$\begin{aligned} {}^{m}\theta_s^b = M\hat{O}'S_b &= M\hat{O}'S_d - S_d\hat{O}'S_b \\ &= {}^{d}\theta_s^m - {}^{m}\Delta_s^b. \end{aligned} \tag{49}$$

Here, ${}^{m}\Delta_s^b$ is the true *bhujāntara* correction of the Sun which is obtained from (48) as follows:

$${}^{m}\Delta_s^b = \frac{{}^{d}\Delta_s^m}{21600'} \times \dot{\theta}_s^t \text{ (min)}, \tag{50}$$

where $\dot{\theta}_s^t$ represents the true rate of motion of the Sun in min/civil day,[104] as observed from O'. The *bhujāntara* correction is applied in a similar manner for the Moon and other planets.

7.1.2 *The bhujāntara correction applied before manda correction*

Having known from (36) the mean longitude (θ_s^d) of the *deśāntara* corrected Sun at the instant (t^d) of mean sunrise at L', indicated by $M\hat{O}S_d$ in Figure 11, the mean longitude (${}^{d}\theta_s^b$) of the Sun (S_b) at the instant (t^b) of true sunrise at L' is obtained by applying the *bhujāntara* correction in the following manner:[105]

$$\begin{aligned} {}^{d}\theta_s^b = M\hat{O}S_b &= M\hat{O}S_d - S_d\hat{O}S_b \\ &= \theta_s^d - {}^{d}\Delta_s^b. \end{aligned} \tag{51}$$

Here, ${}^{d}\Delta_s^b$ is the mean *bhujāntara* correction of the Sun which is obtained from (48) as follows:

$${}^{d}\Delta_s^b = \frac{{}^{d}\Delta_s^m}{21600'} \times \dot{\theta}_s^\circ \text{ (min)}, \tag{52}$$

103 See *Mahābhāskarīya* verse IV.24, Sastri (1957: XC), and Apte (1945: 44).

104 See Section 14.1.6 for our discussion on the true rate of motion of the planets. As already noted in footnote 101, strictly speaking, the units should be in min/sidereal day. However, we have approximated to min/civil day for convenience.

105 See *Mahābhāskarīya* verse IV.7, Sastri (1957: LXXXVIII), Apte (1945: 40), and *Laghubhāskarīya* verses II.4–5, Shukla (1963: 18–19).

where $\dot{\theta}_s^\circ$ represents the mean rate of motion of the Sun in min/civil day,[106] as observed from O, and obtained from (4). Employing (4) in (52), we have

$$^{d}\Delta_s^b = {}^{d}\Delta_s^m \times \frac{59.136 \times 60}{21600} \approx {}^{d}\Delta_s^m \times \frac{3}{18} \text{ (sec)}, \tag{53}$$

which is equivalent to the correction term in (44).

Similarly, (51) and (52) can be extended to the Moon. The mean longitude (${}^{d}\theta_m^b$) of the Moon at the instant (t^b) of true sunrise at L' is obtained by applying *bhujāntara* correction in the following manner:

$$^{d}\theta_m^b = \theta_m^d - {}^{d}\Delta_m^b, \tag{54}$$

where θ_m^d is the mean longitude of the *deśāntara* corrected Moon, obtained from (37). Here, ${}^{d}\Delta_m^b$ is the mean *bhujāntara* correction of the Moon which is obtained from (48) as follows:

$$^{d}\Delta_m^b = \frac{{}^{d}\Delta_s^m}{21600'} \times \dot{\theta}_m^c \text{ (min)}, \tag{55}$$

where $\dot{\theta}_m^c$ represents the corrected mean rate of motion of the Moon in min/civil day,[107] obtained from (16). Employing (16) in (55), we have

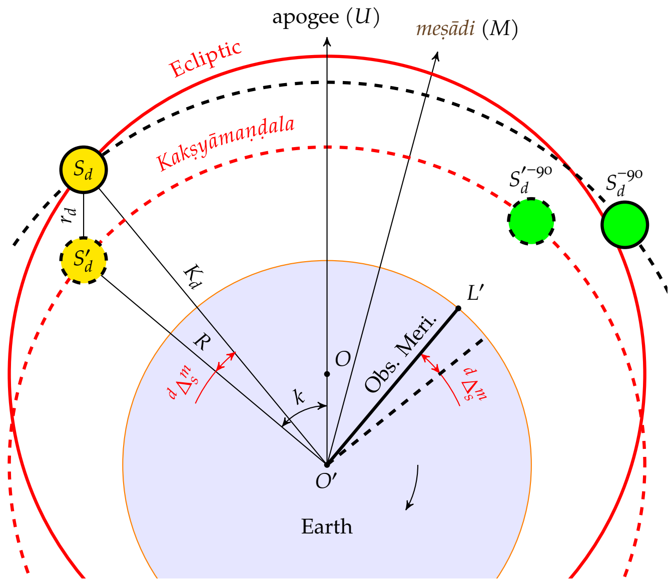
$$^{d}\Delta_m^b = {}^{d}\Delta_s^m \times \frac{790.581}{21600} \approx {}^{d}\Delta_s^m \times \frac{3}{82} \text{ (min)}, \tag{56}$$

which is equivalent to the correction term in (45).

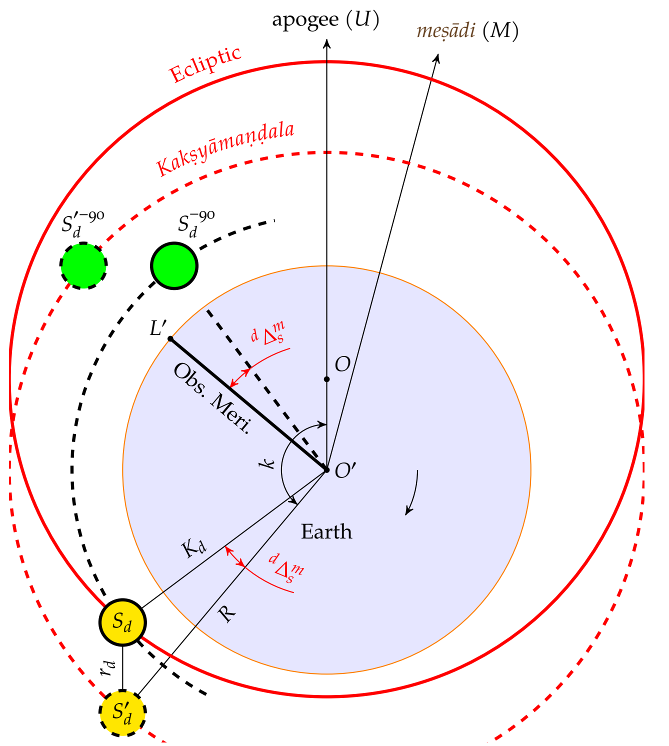
Thus, from (53) and (56), we obtain the mean *bhujāntara* correction of the Sun and the Moon, as stated in verses 8 and 9 of the *Tithinirṇaya*. Applying this correction, as indicated in (44) and (45), results in the mean longitudes of the Sun and the Moon, respectively, at the instant (t^b) of true sunrise at L'. One final correction, known as *manda*, is further applied to obtain the true longitudes (${}^{b}\theta_s^m$ and ${}^{b}\theta_m^m$) of the Sun and the Moon at true sunrise at L'. The computation and application of this correction are given in verse 15 of *Tithinirṇaya*, discussed in Section 11. The application of the *manda* correction to the *bhujāntara* corrected mean Sun ($M\hat{O}S_b = {}^{d}\theta_s^b$) results in the true longitude ($M\hat{O}'S_b = {}^{b}\theta_s^m$) of the Sun at true sunrise at L', which is same as $M\hat{O}'S_b = {}^{m}\theta_s^b$, obtained from (49). Thus, both the approaches, explained in Sections 7.1.1 and 7.1.2, give the same result. Similar to the rationale mentioned in *deśāntara*-correction, the effect of *bhujāntara* is neglected for Moon's apogee.

106 Strictly speaking, the units of the rate of motion should be in min/sidereal day.

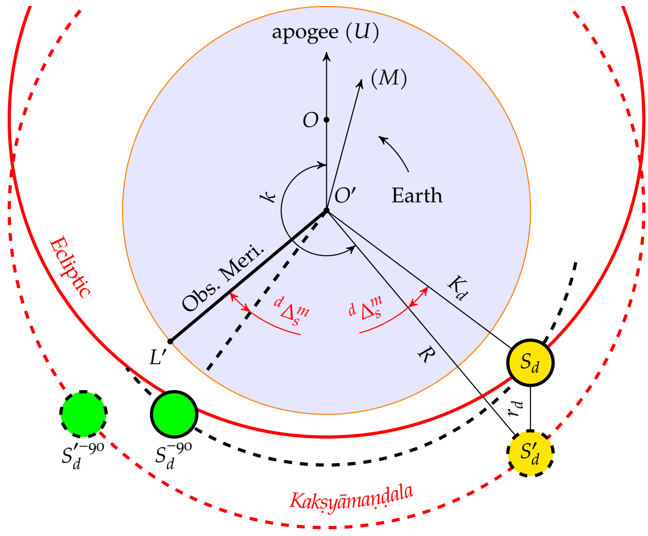
107 Strictly speaking, the units of the rate of motion should be in min/sidereal day.


apogee (U)
meṣādi (M)
Ecliptic
Kakṣyāmaṇḍala
S_d
S'_d
r_d
K_d
R
S'^{-90}_d
S^{-90}_d
L'
Obs. Meri.
O
O'
k
${}^d\Delta^m_s$
Earth

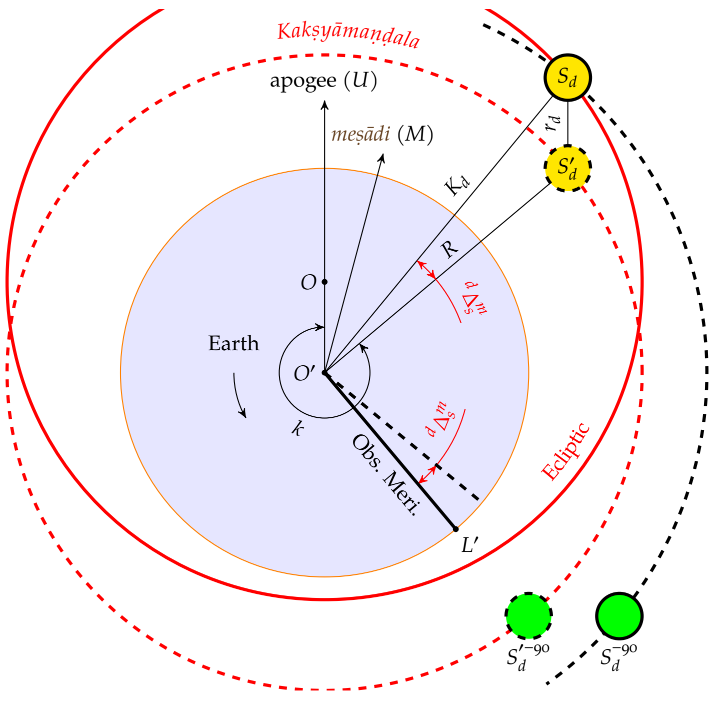
(a)


apogee (U)
meṣādi (M)
Ecliptic
Kakṣyāmaṇḍala
S'^{-90}_d
S^{-90}_d
L'
${}^d\Delta^m_s$
Obs. Meri.
O
O'
k
Earth
K_d
R
S_d
r_d
S'_d

(b)



(c)



(d)

Figure 12: Diagrams showing the rotation of the Earth backward (clockwise) and forward (anticlockwise) in time to arrive at the instant (t^b) of the true sunrise at L' for those positions of the Sun (S_d), whose *kendra* ($k = \theta_s^d - \theta_{s\_ap}$) or anomaly is in (a) first quadrant (b) second quadrant (c) third quadrant, and (d) fourth quadrant.

7.1.3 Sign of the bhujāntara correction

Now, as we understand that the *bhujāntara* correction is done to compute the mean (or true) longitudes of the planets (p) at the instant (t^b) of true sunrise at L', the sign of the *bhujāntara* correction depends on whether the Earth's rotation is performed forward or back in time, to align the observer's meridian with the fictional body S_d^{-90}. When the observer at O' observes the Sun S_d above the horizon, then the Earth has to be rotated back in time, and the *bhujāntara* correction should be subtracted. On the other hand, if the sunrise has not yet occurred for the observer at O', then the Earth has to be rotated forward in time, and the *bhujāntara* correction should be added. The sign of the *bhujāntara* correction can be understood with the help of Figure 12, which is similar to Figure 9, and depicts the Sun (S_d) in different quadrants with respect to its apogee (U) at the instant (t^d) of mean sunrise at L'. In other words, the figure depicts the anomaly or *kendra* ($k = \theta_s^d - \theta_{s\_ap}$) of the Sun in four different quadrants. It is observed from Figures 12a and 12b that, when the *deśāntara* corrected Sun's (S_d) *kendra* is in the first and second quadrants respectively, i.e., $0^\circ \leq \theta_s^d - \theta_{s\_ap} \leq 180^\circ$, the Earth has to be rotated back in time (clockwise) to align the observer's meridian with S_d^{-90}, and thus the *bhujāntara* correction should be subtracted. These two figures correspond to the situation depicted in Figure 10. Similarly, it is observed from Figures 12c and 12d that, when the *kendra* is in the third and fourth quadrants respectively, i.e., $180^\circ \leq \theta_s^d - \theta_{s\_ap} \leq 360^\circ$, the Earth has to be rotated forward in time (anti-clockwise), and thus the *bhujāntara* correction should be added. In these two cases, the observer at O' in Figure 10 would not yet have witnessed sunrise. As will be discussed in Section 11.1, the sign of the *manda* correction of the Sun is also negative when its anomaly is in the first and second quadrants and positive in the third and fourth quadrants. Thus, we find correspondence between the signs of the *bhujāntara* correction and *manda* correction of the Sun, which substantiates the statement in verse 8(a,b).

7.1.4 Approximation of true sunrise

The *bhujāntara* correction of the Sun results in the movement of the Sun from S_d to S_b on the ecliptic by a magnitude of ${}^m\Delta_s^b$ with respect to observer at O', as shown in Figure 11. Thus, the Sun's fictional counterpart also moves from S_d^{-90} to S_b^{-90}, which however is not aligned with the observer's meridian, indicating that the *bhujāntara* corrected Sun (S_b) does not correspond to the instant of true sunrise, by a magnitude of $S_d^{-90}\hat{O}'S_b^{-90} = S_d\hat{O}'S_b = {}^m\Delta_s^b$. This error was perhaps considered small. Possibly, a second iteration of the *bhujāntara* correction, involving rotating the Earth further by ${}^m\Delta_s^b$, could result in an even more precise estimation of the instant of true sunrise at L', if required.

7.1.5 *Anomalies in the interpretation of the sequence of bhujāntara and manda corrections*

Following our discussion in Sections 7.1.1 and 7.1.2 it is apparent that the *bhujāntara* correction should be performed after the *manda* correction only if the true rate of motion of the planet is known. Otherwise, the *bhujāntara* correction should be performed before the *manda* correction. The true rate of motion of the planet varies from instant to instant whereas the mean rate of motion of the planet is always constant and known from the *Āryabhaṭīya* parameters of a *mahāyuga*. As the *bhujāntara* corrections for the Sun and the Moon proposed in the *Tithinirṇaya*, in (44) and (45) respectively, have constant multipliers and divisors, it is clear that the text adopts the mean rate of motion of the planets for computation as discussed in Section 7.1.2. Thus, this implies that the *bhujāntara* correction should always be applied before the *manda* correction as per the procedure of the *Tithinirṇaya*.[108] However, in the course of working out an example, Bannañje (1974*b*: 186–187) and Vyāsadāsa (2007: 26–31) apply the *bhujāntara* correction after *manda* for the Sun and before *manda* for the Moon.[109] In our opinion, the sequence of corrections in the former case is only valid if the true rate of motion of the Sun is employed.[110]

Finally, we would like to note that some texts such as *Khaṇḍakhādyaka* and *Karaṇaratna* apply the *bhujāntara* correction for the Moon but not for the Sun.[111] This results in approximating the true longitude of the Sun at the instant of true sunrise.

8 RSINE VALUES OF 24 ARCS

शरीरनुत् धीभवनः कथञ्चनो
नळीजनो मानपटुः शुकालपः । [112]
निरामयो धीःपथिको नृपाधिको
बुधोनरः सुप्तखरः कलाविराट् ॥ १० ॥ [113]

108 Even verse 22(a,b) of *Tithinirṇaya* states that the *deśāntara* and *bhujāntara* corrections are applied to the mean planet.

109 The same sequence is proposed by K. S. Shukla while commenting *Laghubhāskarīya*, Shukla (1963: 27–28), and *Mahābhāskarīya*, Shukla (1960: 129–130).

110 See the commentary of Pṛthūdakasvāmin on *Brāhmasphuṭasiddhānta* verse II.29, R. S. Sharma (1966: 197), and Parameśvara on *Mahābhāskarīya* verses IV.7, 24, Apte (1945: 40,44).

111 See *Khaṇḍakhādyaka* verse I.18, Sengupta (1934), and *Karaṇaratna* verse I.26 (b,d), Shukla (1979: 19–20).

112 Bannañje (1974*b*: 183) notes an alternate reading शरीररद्धीभवनः as a scribal error. Bhikṣu (n.d.) has the reading शरीरनूर्ध्वंभवनः कथञ्चोनो नधीजनो मानपटुः शुकालयः।

113 Bannañje (1974*b*: 183) notes an alternate reading निरालयो as a scribal error. He further notes that the phrase नृपाधिको, which is not present in the manuscript, has been reconstructed by him as per the required value in the Sine Table. Also, he suggests सुप्तखरः would match the numeral instead सुप्तपरः in the manuscript. Bhikṣu (n.d.) has the reading धीपथिको नयाधिको..।

महाशरो दूरसरो धमीहरिः
हसन्धुरो वेदनगः सुसङ्कुलः । [114]
तमःखगः पारबलं रसोबली
धनावलिः कालभृगुर्जगद्भगः ॥ ११ ॥ [115] ॥ वंशस्थ ॥
इमाश्चतुर्विंशतिज्याः स्फुटत्वायार्कसोमयोः ।
चतुर्विंशतिवाक्यानि त्रिराशीनामिमान् विदुः ॥ १२ ॥ [116] ॥ अनुष्टुभ् ॥

śarīranut dhībhavanaḥ kathañcano
naḷījano mānapaṭuḥ śukālapaḥ |
nirāmayo dhīḥpathiko nṛpādhiko
budhonaraḥ suptakharaḥ kalāvirāṭ || 10 ||
mahāśaro dūrasaro dhamīhariḥ
hasandhuro vedanagaḥ susaṅkulaḥ |
tamaḥkhagaḥ pārabalaṃ rasobalī
dhanāvaliḥ kālabhṛgurjagadbhagaḥ || 11 || || *vaṃśastha* ||
imāścaturviṃśatijyāḥ sphuṭatvāyārkasomayoḥ |
caturviṃśativākyāni trirāśīnāmimān viduḥ || 12 || || *anuṣṭubh* ||

Śarīranut (225), *dhībhavana* (449), *kathañcana* (671), *naḷījana* (890), *mānapaṭu* (1105), *śukālapa* (1315), *nirāmaya* (1520), *dhīḥpathika* (1719), *nṛpādhika* (1910), *budhonara* (2093), *suptakhara* (2267), *kalāvirāṭ* (2431), *mahāśara* (2585), *dūrasara* (2728), *dhamīhari* (2859), *hasandhura* (2978), *vedanaga* (3084), *susaṅkula* (3177), *tamaḥkhaga* (3256), *pārabala* (3321), *rasobalī* (3372), *dhanāvali* (3409), *kālabhṛgu* (3431), *jagadbhaga* (3438). [Scholars] knew these 24 Rsine values of a quadrant (*trirāśi*) as [stated in the form of] these 24 *vākyas* for [obtaining] the trueness (true longitudes) of the Sun and the Moon.

In order to compute the equation of center of a planet, the Rsine of its anomaly (*kendra*) has to be determined. Hence, in the above verses, the author gives the values of twenty-four Rsines in minutes,[117] corresponding to the 24 arcs obtained by dividing the quadrant (3 *rāśi*s) into 24 parts of 225′ each. The Rsine values stated in the above verses are summarized in Table 4. These Rsine values can

114 Bannañje (1974*b*: 183) notes an alternate reading वमी हरिः as a scribal error. Bhikṣu (n.d.) has the reading दुरसरो धमाहरिः हसन्धरो..।

115 Bannañje (1974*b*: 183) notes an alternate reading नमःखगः as a scribal error and the phrase रसोबली, which is not present in the manuscript was constructed as per the numeral. Bhikṣu (n.d.) has the reading रसाबलम्।

116 This half verse is missing in Bhikṣu (n.d.). Also, the phrase should be इमानि instead of इमान्। This appears to be an incorrect usage or exercise of poetic license for metrical considerations.

117 The circumference of a circle is considered to be ($360^\circ \times 60 =$) 21600′. Hence, the radius of the circle (R) would be $\frac{21600}{2\pi} \approx 3438'$.

be obtained by employing the Rsine difference values given by Āryabhaṭa in the *Āryabhaṭīya*.[118]

It is worth noting that verses similar to 10–11 are found in the earlier works such as Śaṅkaranārāyaṇa's commentary of *Laghubhāskarīya*, and in later works such as *Grahaṇamaṇḍana* of Parameśvara, and *Uparāgakriyākrama* of Acyuta Piṣāraṭi.[119]

118 See *Āryabhaṭīya* verse 12 in the *Gītikā* chapter, Shukla and Sarma (1976: 29–30). Also, see *Śiṣyadhīvṛddhidatantra* verses II.1–4, Chatterjee (1981: 34).

119 See Śaṅkaranārāyaṇa's commentary on *Laghubhāskarīya* verses II.2–3, S. Jhā (2007), *Grahaṇamaṇḍana* verses 25(A,B), Sarma (1977: 11), *Uparāgakriyākrama* verses I.19–21, Piṣāraṭi (n.d.).

S.No.	Arc		Rsine (in min)		
			in *Tithinirṇaya*		computed
	$x^\circ y'$	minutes	phrase	value	
1	$3^\circ 45'$	225	*śarīranut*	225	224.839
2	$7^\circ 30'$	450	*dhībhavana*	449	448.716
3	$11^\circ 15'$	675	*kathañcana*	671	670.671
4	$15^\circ 00'$	900	*naḷījana*	890	889.754
5	$18^\circ 45'$	1125	*mānapaṭu*	1105	1105.027
6	$22^\circ 30'$	1350	*śukālapa*	1315	1315.569
7	$26^\circ 15'$	1575	*nirāmaya*	1520	1520.476
8	$30^\circ 00'$	1800	*dhīḥpathika*	1719	1718.873
9	$33^\circ 45'$	2025	*nṛpādhika*	1910	1909.910
10	$37^\circ 30'$	2250	*budhonara*	2093	2092.768
11	$41^\circ 15'$	2475	*suptakhara*	2267	2266.664
12	$45^\circ 00'$	2700	*kalāvirāṭ*	2431	2430.854
13	$48^\circ 45'$	2925	*mahāśara*	2585	2584.635
14	$52^\circ 30'$	3150	*dūrasara*	2728	2727.348
15	$56^\circ 15'$	3375	*dhamīhari*	2859	2858.382
16	$60^\circ 00'$	3600	*hasandhura*	2978	2977.176
17	$63^\circ 45'$	3825	*vedanaga*	3084	3083.221
18	$67^\circ 30'$	4050	*susaṅkula*	3177	3176.064
19	$71^\circ 15'$	4275	*tamaḥkhaga*	3256	3255.306
20	$75^\circ 00'$	4500	*pārabala*	3321	3320.608
21	$78^\circ 45'$	4725	*rasobalī*	3372	3371.691
22	$82^\circ 30'$	4950	*dhanāvali*	3409	3408.336
23	$86^\circ 15'$	5175	*kālabhṛgu*	3431	3430.386
24	$90^\circ 00'$	5400	*jagadbhaga*	3438	3437.747

Table 4: Rsine values in minutes given in the *Tithinirṇaya*.

9 INTERPOLATION FORMULA FOR OBTAINING THE DESIRED RSINE

शुभाङ्गपरिमाणेन यदि ज्यार्धं न पूर्यते ।[120]
वर्तमानज्यया हत्वा मुरारिफलसङ्ग्रहः ॥ १३ ॥[121] ॥ अनुष्टुभ् ॥

śubhāṅgaparimāṇena yadi jyārdhaṃ na pūryate |
vartamānajyayā hatvā murāriphalasaṅgrahaḥ || 13 || || *anuṣṭubh* ||

If [the *kendra* in minutes] is not exhausted (*na pūryate*) by [the multiples of] *śubhāṅga* (3°45′ = 225′), having multiplied [the remaining minutes (*kalā-śeṣa*)] by the current Rsine [difference] ([*śiṣṭa*]-*vartamānajyā*),[122] the result upon dividing by *murāri* (225) [when] added [to the elapsed Rsine (*gata-jyā*), is the desired] Rsine ([*iṣṭa*]-*jyārdha*).[123]

The above verse prescribes the following interpolation formula for obtaining the desired Rsine (*iṣṭa-jyā*) of the *kendra*:[124]

$$\textit{iṣṭa-jyā} = \textit{gata-jyā} + \frac{\textit{śiṣṭa-vartamānajyā} \times \textit{kalā-śeṣa}}{\textit{murāri}}. \tag{57}$$

9.1 EXPLANATION

Given, from Table 4, the Rsine values in minutes for every 225′ interval of *kendra*, the Rsine value of any desired *kendra* which lies within any given interval is computed with the help of the interpolation formula as follows: If $R\sin(k_i)$ and $R\sin(k_{i+1})$ are the Rsine values corresponding to the successive values of *kendra*, k_i and k_{i+1} respectively, then the desired Rsine ($R\sin(k_j)$) corresponding to k_j, which lies in between k_i and k_{i+1}, is given by:

$$R\sin(k_j) = R\sin(k_i) + \frac{[R\sin(k_{i+1}) - R\sin(k_i)] \times \left(k_j - k_i\right)}{225}, \tag{58}$$

which is the same as (57) described in the verse.

120 Bhikṣu (n.d.) has the reading शुभ्राग्र that denotes 225′ and is equivalent to the reading शुभाङ्ग which denotes 3°45′.

121 Bannañje (1974b: 184) opines that though all manuscripts contain हत्वा, it should be read as हित्वा and mentions यदि ज्यार्थम् to be scribal error.

122 Vyāsadāsa (2007: 22) notes the use of *śiṣṭa* in order to describe the meaning of this verse.

123 In Indian astronomy, the phrases *jyā* (chord) and *jyārdha* (semi-chord) are used interchangeably to represent Rsine of an arc.

124 See *Śiṣyadhīvṛddhidatantra* verse II.12, Chatterjee (1981: 36).

10 QUADRANTS OF ECLIPTIC AND *BHUJA*

राशिचक्रं चतुष्पादमोजानोजद्विपादयोः ।
अतीतानागतौ भागौ भुज इत्युच्यते बुधैः ॥ १४ ॥ [125] ॥ अनुष्टुभ् ॥

rāśicakraṃ catuṣpādamojānojadvipādayoḥ |
atītānāgatau bhāgau bhuja ityucyate budhaiḥ || 14 || || *anuṣṭubh* ||

The circle (*rāśicakra*) of *rāśis* [consisting] of four quadrants (*catuṣpāda*) [is considered]. The [arc in] degrees traversed and yet to be traversed in the two odd (*oja*) and even (*anoja*) quadrants [respectively] is said to be the *bhuja* by the intelligent.

The above verse intends to prescribe a method to compute the desired Rsine of the arcs that belong to different quadrants of a circle. The verse suggests grouping the quadrants of a circle, or *rāśicakra*, into odd (*oja*) and even (*anoja*). Further, the verse introduces the term *bhuja*, defining it as the angle in degrees traversed in the odd quadrants and yet to be traversed in the even quadrants. Although the verse does not explicitly state it, the magnitude of the Rsine of the arc is the Rsine of *bhuja*.

10.1 EXPLANATION

Sections 8 and 9 outline a procedure for computing the desired Rsine specifically for arcs in the first quadrant. This section addresses the process of obtaining the desired Rsine for arcs in other quadrants. In Indian astronomy, the *rāśicakra* is also used to represent degrees in a circle,[126] which can be understood with the help of Figure 13. Figure 13a depicts a circle, where 0° is indicated by *meṣādi* and each quadrant of the circle is constituted of three *rāśis* of 30° each. These quadrants, as described in the verse, are grouped into odd (*oja*) and even (*anoja*) quadrants. To compute the Rsines of arcs, which belong to different quadrants, the verse defines a term named *bhuja*.[127] The *bhuja* is the angle traversed in the odd quadrants and the angle yet to be traversed in the even quadrants and can be understood with the help of Figure 13b.

Figure 13b is similar to Figure 13a and depicts a circle of four quadrants indicated by Q_1, Q_2, Q_3, and Q_4. As the application of Rsines in *Tithinirṇaya* is in the computation of the equation of center of the Sun and the Moon, the arcs of the

125 Bannañje (1974*b*: 185) notes अतीतानागतौ पादौ as an alternate reading.

126 This *rāśicakra* should not be confused with the ecliptic with Zodiac signs (*rāśis*) in the background.

127 See *Laghubhāskarīya* verses II.1–2(a,b), Shukla (1963: 16), *Mahābhāskarīya* verse IV.8, Shukla (1960: 115) , *Karaṇaratna* verses A.28–29, Shukla (1979: 111), *Śiṣyadhīvṛddhidatantra* verse II.11, Chatterjee (1981: 36), *Laghumānasa* verses III.1–2, Shukla (1990: 120–121).

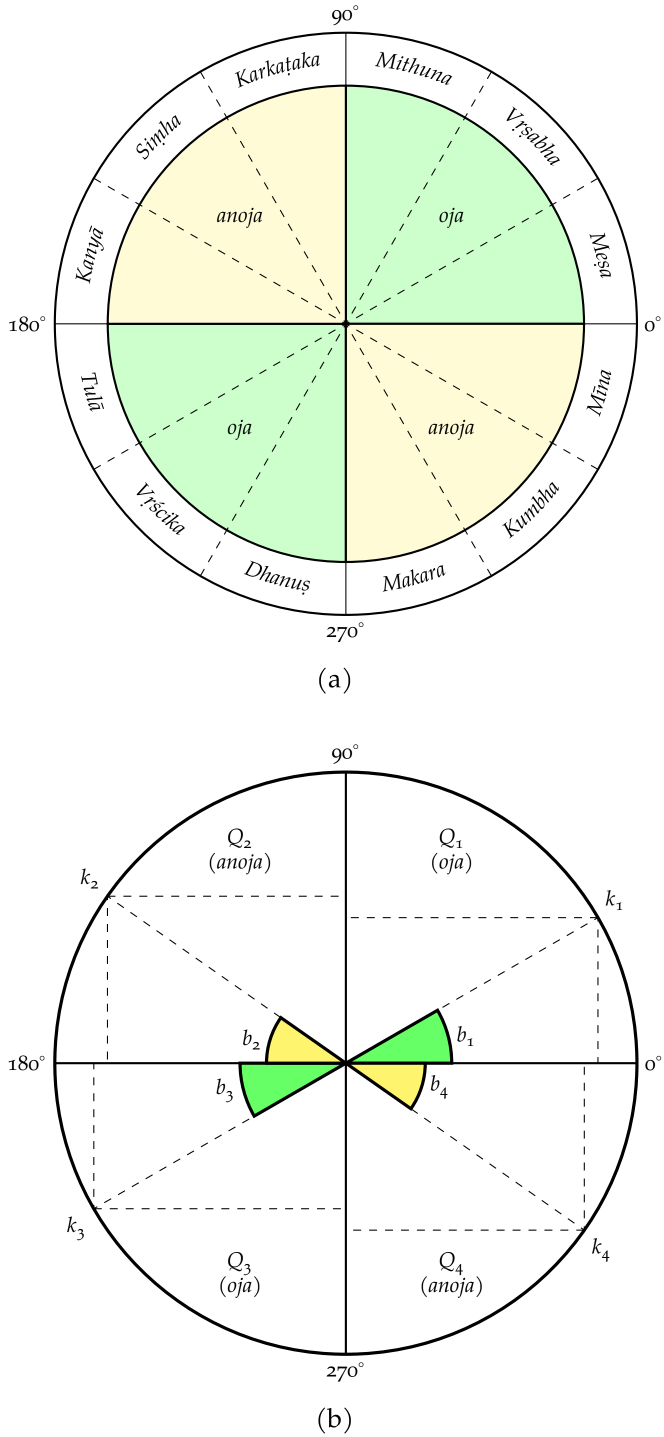


Figure 13: (a) A diagram indicating the division of *rāśicakra* into odd (*oja*) and even (*anoja*) quadrants and (b) A diagram depicting *bhuja* in each quadrant.

circle represent the *kendras* (anomalies) of the Sun and the Moon. The *kendras* of the Sun (${}^{d}\theta_{s}^{b} - \theta_{s\_ap}$), obtained from (44) and (43), and the Moon (${}^{d}\theta_{m}^{b} - \theta^{\circ}_{m\_ap}$), obtained from (45) and (23), respectively, could fall in any one of the four quadrants of the circle. If k_1, k_2, k_3, and k_4 are the *kendras* which fall in quadrants Q_1, Q_2, Q_3, and Q_4 respectively, then their corresponding *bhujas* will be $b_1 = k_1$, $b_2 = 180^\circ - k_2$, $b_3 = k_3 - 180^\circ$, and $b_4 = 360^\circ - k_4$ respectively. Further, the desired Rsine of *bhuja*, where $0^\circ \leq$ *bhuja* $\leq 90^\circ$, is computed from the procedure given in Sections 8 and 9. It is observed that the magnitude of the Rsine of the *kendra* is the same as the Rsine of the *bhuja* and can be understood from the following relations:

$$|R\sin(kendra)| = \begin{cases} |R\sin(k_1)| = R\sin(b_1) & , 0^\circ \leq k_1 \leq 90^\circ \\ |R\sin(k_2)| = R\sin(180^\circ - b_2) = R\sin(b_2) & , 90^\circ \leq k_2 \leq 180^\circ \\ \left|R\sin(k_3)\right| = \left|R\sin(180^\circ + b_3)\right| = R\sin(b_3) & , 180^\circ \leq k_3 \leq 270^\circ \\ \left|R\sin(k_4)\right| = \left|R\sin(360^\circ - b_4)\right| = R\sin(b_4) & , 270^\circ \leq k_4 \leq 360^\circ \end{cases}$$

11 *MANDA* CORRECTION: TO OBTAIN TRUE LONGITUDES AT TRUE SUNRISE AT THE OBSERVER'S MERIDIAN

स्वोच्चोनार्काब्जयोः दोर्ज्या गोसद्भ्यां वर्धिताः क्रमात् ।[128]
अजलब्धकलाः स्वस्वगोळयोः स्याद्दृणं धनम् ॥ १५ ॥[129] ॥ अनुष्टुभ् ॥

svocconārkābjayoḥ dorjyā gosadbhyāṃ vardhitāḥ kramāt |
ajalabdhakalāḥ svasvagoḷayoḥ syādṛṇaṃ dhanam || 15 || || *anuṣṭubh* ||

The Rsines, [having the longitudes] of the Sun and the Moon reduced by their own apogees, are multiplied by *go* (3) and *sad* (7) respectively. The minutes obtained upon dividing by *aja* (80) shall be [applied to their mean longitudes] negatively [or] positively in their (Sun's and Moon's) respective hemispheres.[130]

The above verse prescribes the *manda* correction (${}^{b}\Delta_{s}^{m}$ and ${}^{b}\Delta_{m}^{m}$), or the equation of center, for the *bhujāntara* corrected mean Sun (${}^{d}\theta_{s}^{b}$) and mean Moon (${}^{d}\theta_{m}^{b}$), which is required due to the eccentricity of their respective orbits. This correction results in the true longitudes of the Sun (${}^{b}\theta_{s}^{m}$) and the Moon (${}^{b}\theta_{m}^{m}$) at the

128 Bannañje (1974*b*: 185) notes that the alternate reading सोच्चोनार्का is a scribal error.
129 We have considered the reading स्याद्दृणं धनम् from Bhikṣu (n.d.) whereas Bannañje (1974*b*: 185) has the reading सत्रदृणं धनम्..।
130 In *Karaṇaratna* verse I.38(a,b), Shukla (1979: 28), we find that the word *gola* is used in the sense of hemisphere, where the northern and southern hemispheres are denoted by the terms *uttara-gola* and *dakṣiṇa-gola* respectively. Each of them corresponds to the six *rāśis* beginning from the first point of Aries ($0^\circ - 180^\circ$) and Libra ($180^\circ - 360^\circ$) respectively.

instant (t^b) of true sunrise at L'. The following relations are prescribed in the above verse:[131]

$$
\begin{aligned}
{}^{b}\theta_s^m = {}^{d}\theta_s^b \mp \left|{}^{b}\Delta_s^m\right| &= {}^{d}\theta_s^b \mp \left|\frac{go}{aja} \times R\sin({}^{d}\theta_s^b - \theta_{s\_ap})\right| \text{ (in } \textit{kalā}\text{)} \\
&= {}^{d}\theta_s^b \mp \left|\frac{3}{80} \times R\sin({}^{d}\theta_s^b - \theta_{s\_ap})\right| \text{ (in min)} \qquad (59)
\end{aligned}
$$

$$
\begin{aligned}
{}^{b}\theta_m^m = {}^{d}\theta_m^b \mp \left|{}^{b}\Delta_m^m\right| &= {}^{d}\theta_m^b \mp \left|\frac{sad}{aja} \times R\sin({}^{d}\theta_m^b - \theta^{\circ}_{m\_ap})\right| \text{ (in } \textit{kalā}\text{)} \\
&= {}^{d}\theta_m^b \mp \left|\frac{7}{80} \times R\sin({}^{d}\theta_m^b - \theta^{\circ}_{m\_ap})\right| \text{ (in min),} \qquad (60)
\end{aligned}
$$

where $({}^{d}\theta_s^b - \theta_{s\_ap})$ and $({}^{d}\theta_m^b - \theta^{\circ}_{m\_ap})$ are the anomalies (*kendra*s) of the *bhujāntara* corrected Sun and Moon respectively. The verse notes that the correction is negative for those anomalies in the northern hemisphere (first and second quadrants), and positive for those in the southern hemisphere (third and fourth quadrants).

11.1 EXPLANATION

In Section 7.1, we examined the effects of displacing the true observer (O') from the center (O) of the orbit of the Sun. This section generalizes the impact of displacing the true observer from the center of the orbit of any given planet P. The *manda* correction (equation of center) in Indian astronomy encapsulates this impact and its geometric rationale can be understood with the help of Figure 14. This figure depicts a planet (P) orbiting in the *grahabhramaṇavṛtta* or *pratimaṇḍala* (orbit of the planet) with the mean rate of motion ($\dot{\theta}^{\circ}_p$ or $\dot{\theta}^{c}_p$).[132] The planet's orbit is centered at O, with radius $OP = R$. The figure also depicts the mean longitudes of the planet (θ_p) and its apogee (θ_{p_ap}) indicated by $M\hat{O}P$ and $M\hat{O}U$ respectively.[133] Further, consider an observer at O',[134] at a distance r from the orbit's center (O) in the direction opposite to the apogee (U). Now, with respect

131 See *Laghubhāskarīya* verses II.3(c,d)–4(a,b), Shukla (1963: 18), *Mahābhāskarīya* verses IV.4(c,d)–6, Shukla (1960: 110–111), *Śiṣyadhīvṛddhidatantra* verse II.14, Chatterjee (1981: 37), *Tantrasaṅgraha* verses II.21–22, 35–36, Ramasubramanian and Sriram (2011: 75–76,89–90).

132 The mean rate of motion is the rate of motion of the planet (P) with respect to the observer at orbit's center (O). The rates of motion of the Sun ($\dot{\theta}^{\circ}_s$) and the Moon ($\dot{\theta}^{c}_m$) are obtained from (4) and (16) respectively.

133 Here, in case of the Sun and the Moon, the mean planet would be the *bhujāntara* corrected mean Sun (${}^{d}\theta_s^b$) and the *bhujāntara* corrected mean Moon (${}^{d}\theta_m^b$), as obtained from (44) and (45), respectively. The apogees for the Sun and the Moon are θ_{s_ap} and $\theta^{\circ}_{m\_ap}$, as obtained from (43) and (23), respectively.

134 The Earth is also positioned according to the position of the observer.

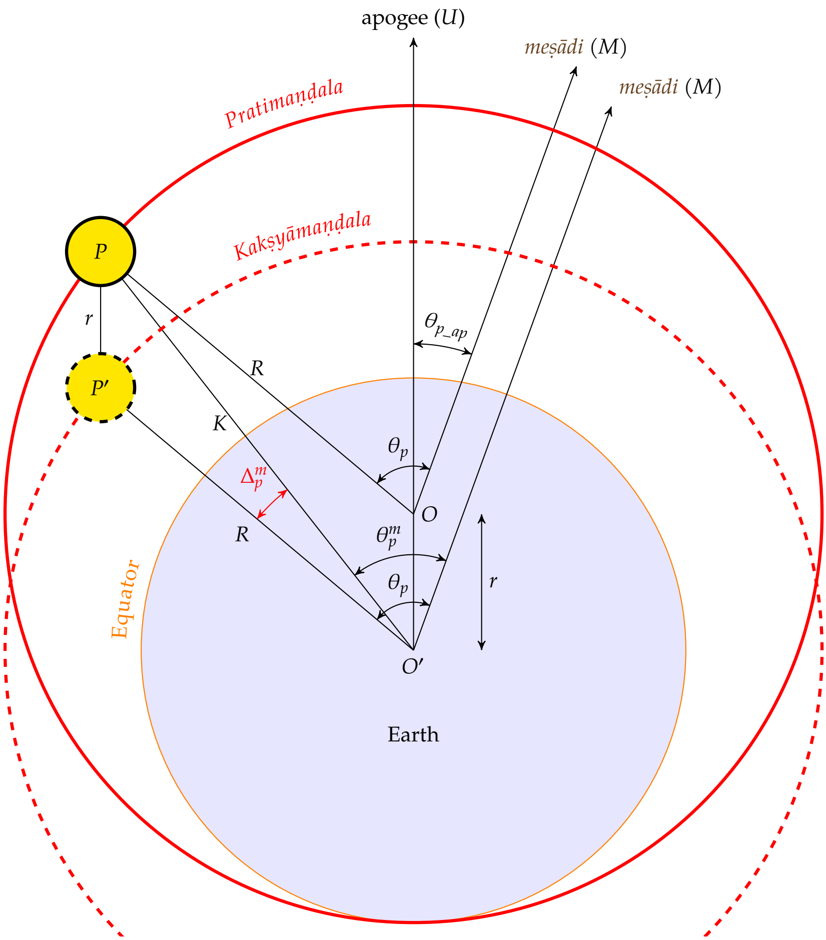


Figure 14: A diagram showing the *manda* correction (Δ_p^m) of a planet (P) due to the eccentricity (r) of the orbit.

to *meṣādi* (M), the observer at O' views the planet P at an angle $M\hat{O}'P = \theta_p^m$, which is the true longitude of the planet, situated at a distance $O'P = K$, known as *manda*-*karṇa*.

To compute the true longitude of the planet (θ_p^m), consider a fictitious mean planet P', orbiting in the *kakṣyāmaṇḍala*,[135] having the mean longitude $M\hat{O}'P' =$

135 A fictitious orbit centered at O', having the same radius ($O'P' = R$) as the *pratimaṇḍala*, and displaced from its center by a distance $OO' = PP' = r$.

θ_p. It can be easily seen that the true longitude of the planet (θ_p^m) is given by[136]

$$\begin{aligned}\theta_p^m = M\hat{O}'P &= M\hat{O}'P' - P\hat{O}'P' \\ &= \theta_p - \Delta_p^m \\ &= \theta_p - \sin^{-1}\left(\frac{r}{K} \times \sin(\theta_p - \theta_{p\_ap})\right), \qquad (61)\end{aligned}$$

where Δ_p^m and θ_{p_ap} are the planet's equation of center and the mean longitude of the planet's apogee respectively.

Now, consider the *manda* correction for the *bhujāntara* corrected mean Sun (${}^d\theta_s^b$) and mean moon (${}^d\theta_m^b$). Substituting the *bhujāntara* corrected mean Sun (${}^d\theta_s^b$) and mean moon (${}^d\theta_m^b$) obtained from (44) and (45) in place of the mean planet (θ_p) in (61), the Sun's apogee (θ_{s_ap}) and the Moon's apogee ($\theta^\circ_{m\_ap}$) obtained from (43) and (23) in place of the planet's apogee (θ_{p_ap}), and applying the ratios $\frac{r}{K} = \frac{r_\circ}{R}$ for the Sun and the Moon to be $\frac{3}{80}$ and $\frac{7}{80}$ respectively as per the Āryabhaṭa school,[137] we obtain the true longitudes of the *bhujāntara* corrected Sun (${}^b\theta_s^m$) and Moon (${}^b\theta_m^m$) to be

$${}^b\theta_s^m = {}^d\theta_s^b - {}^b\Delta_s^m = {}^d\theta_s^b - \sin^{-1}\left(\frac{3}{80} \times \sin({}^d\theta_s^b - \theta_{s\_ap})\right) \qquad (62)$$

$${}^b\theta_m^m = {}^d\theta_m^b - {}^b\Delta_m^m = {}^d\theta_m^b - \sin^{-1}\left(\frac{7}{80} \times \sin({}^d\theta_m^b - \theta^\circ_{m\_ap})\right), \qquad (63)$$

which are equivalent to (59) and (60) respectively. It is evident from (61) that, as stated in the verse, the sign of the correction is negative for $0^\circ < (\theta_p - \theta_{p\_ap}) < 180^\circ$ and positive for $180^\circ < (\theta_p - \theta_{p\_ap}) < 360^\circ$, because the sine function is positive in first and second quadrants and negative in third and fourth quadrants.

12 TREPIDATION OF THE EQUINOX

कल्यब्दौघो धेनुभवो युक्तः सौरैर्वृथाफलैः ।[138]
एतस्मान्मापतेर्लब्धं राश्याद्ययनमुच्यते ॥ १६ ॥[139]
प्रभारत्नं धीसवनं गानस्थानं जनेधनम् ।

[136] See *Gaṇita-yukti-bhāṣā* sections VIII.3–7, Sarma (2008: 622–628), *Tantrasaṅgraha* Appendix F.1.2, Ramasubramanian and Sriram (2011: 492–494), for the derivation.

[137] See *Āryabhaṭīya* verse 10 in the *Gītikā* chapter, Shukla and Sarma (1976: 22–23).

[138] Before this verse, Bannañje (1974*b*: 188), and Bhikṣu (n.d.) have the half verse तदेकदिनगा लिप्ता ग्रहाणां स्वस्वभुक्तयः that is repeated in twenty-first verse. Bannañje (1974*b*: 188) states this to be scribal error.

[139] Bannañje (1974*b*: 188) uses राश्याद्यायनम् whereas Bhikṣu (n.d.) has the reading राश्याद्ययनम्। The latter seems to be more appropriate as the word *ayana* is employed in other astronomical texts. See *Karaṇaratna* verse I.36, Shukla (1979: 25).

देहिनित्यं सुगप्रायं सावलोक्यं तटिद्वपुः ॥ १७ ॥ [140]
नवभार्येति वाक्यानि ज्ञोऽनन्तोऽत्र तु हारकः ।
तद्दोर्ज्यालिप्तिका भानौ युक्त्वा त्यक्त्वाऽथ गोळयोः ॥ १८ ॥ [141] ॥ अनुष्टुभ् ॥

kalyabdaugho dhenubhavo yuktaḥ saurairvṛthāphalaiḥ |
etasmānmāpaterlabdhaṃ rāśyādyayanamucyate || 16 ||
prabhāratnaṃ dhīsavanaṃ gānasthānaṃ janedhanam |
dehinityaṃ sugaprāyaṃ sāvalokyaṃ taṭidvapuḥ || 17 ||
navabhāryeti vākyāni jño'nanto'tra tu hārakaḥ |
taddorjyāliptikā bhānau yuktvā tyaktvā'tha goḷayoḥ || 18 || || *anuṣṭubh* ||

Dhenubhava (4409) added with *sauravṛthāphala* is the collection of *kali* years [elapsed]. The *rāśis*, etc., obtained from the division of this [group of *kali* years] by *māpati* (615) is called *ayana*. *Prabhāratna* (242), *dhīsavana* (479), *gānasthāna* (703), *janedhana* (908), *dehinitya* (1088), *sugaprāya* (1237), *sāvalokya* (1347), *taṭidvapu* (1416), *navabhāryā* (1440) — thus are the *vākyas*, and *jño'nanta* (600) is the divisor here [for interpolation]. Thereafter, in [the true longitude of] the Sun, that Rsine [in] minutes is added or subtracted [if the *ayana* is] in the two (southern and northern) hemispheres [respectively].

The above verses prescribe the procedure to find the *sāyana* longitude[142] of the Sun from its *nirayana* longitude.[143] The *sāyana* longitude is necessary for *udayāntara* and *cara* corrections and can be determined by computing the motion of the vernal equinox (Γ) with respect to *meṣādi* (*M*). The model considered in *Tithinirṇaya* to compute the motion of the equinox is same as the model described in *Karaṇaratna* of Devācārya, and in the commentary of Āmarāja on *Khaṇḍakhādyaka*.[144] To this end, the above verses initially prescribe the determination of a quantity named *ayana* ($\bar{A}$) as follows:

$$\begin{aligned} ayana(\bar{A}) &= \left[\frac{kalyabdaugha}{māpati}\right] = \left[\frac{dhenubhava + sauravṛthāphala}{māpati}\right](\textit{rāśi}, \text{etc.}) \\ &= \left[\frac{\textit{kali}\text{ years}}{615}\right] = \left[\frac{4409 + sauravṛthāphala}{615}\right](\text{signs, etc.}), \end{aligned} \tag{64}$$

140 Bannañje (1974b: 188) notes the alternate readings धीवसनम्, and सुप्रमयम् as scribal errors. He also makes an observation that this verse matches with *Karaṇaratna* verses I.49(c,d)–50(a,b), Shukla (1979: 34–35). Though Bhikṣu (n.d.) does not feature this verse, it contains the commentary of the same.

141 Bhikṣu (n.d.) has the reading परं लवादयो भानोः युक्तास्त्यक्त्वाऽथगोलयोः।

142 The longitude measured with respect to the vernal equinox (Γ).

143 The longitude measured with respect to *meṣādi* (*M*).

144 See *Karaṇaratna* verse I.36, Shukla (1979: 25–26), and the commentary of Āmarāja on *Khaṇḍakhādyaka* verse III.11, Misra (1925: 105–107).

where 4409 and *sauravṛthāphala*[145] are the elapsed *kali* years till epoch and since the epoch respectively.

S. No.	*bhuja* of *ayana* ($\bar{A}$)		Motion of equinox (θ_Γ)		Computed declination (θ_Γ) of C_Γ in min
	in degrees	in min	phrase	in min	
1	10	600	*prabhāratna*	242	243.01
2	20	1200	*dhīsavana*	479	479.79
3	30	1800	*gānasthāna*	703	704.04
4	40	2400	*janedhana*	908	909.35
5	50	3000	*dehinitya*	1088	1089.26
6	60	3600	*sugaprāya*	1237	1237.48
7	70	4200	*sāvalokya*	1347	1348.23
8	80	4800	*taṭidvapu*	1416	1416.78
9	90	5400	*navabhāryā*	1440	1440

Table 5: The motion of vernal equinox (θ_Γ) in minutes corresponding to *bhuja* of *ayana* values.

The above verses, through the phrases *prabhāratna*, etc., further provide the motion of the equinox (θ_Γ) for every 600′ interval of *ayana* ($\bar{A}$) as summarized in Table 5. It is worth noting that verse 17 is also found in *Karaṇaratna*, a seventh century CE astronomical text.[146]

To determine the motion (θ_Γ) of the equinox corresponding to an *ayana* ($\bar{A}$) value which lies within any given interval, the verses hint at an interpolation formula. If $(\theta_\Gamma)_i$ and $(\theta_\Gamma)_{i+1}$ are the motion of the equinox corresponding to the successive values of *ayana*, $\bar{A}_i$ and $\bar{A}_{i+1}$ respectively, then the desired motion of the equinox (θ_Γ) corresponding to $\bar{A}_j$, which lies in between $\bar{A}_i$ and $\bar{A}_{i+1}$, can be obtained by the following interpolation[147]

$$\theta_\Gamma = (\theta_\Gamma)_i + \frac{(\theta_\Gamma)_{i+1} - (\theta_\Gamma)_i}{600'} \times \left(\bar{A}_j - \bar{A}_i\right). \qquad (65)$$

Finally, the above verses derive the *sāyana* longitude of the Sun (λ_s) by applying the motion of equinox (θ_Γ) to the true *nirayana* longitude (${}^b\theta_s^m$) of the *bhujāntara* corrected Sun (S_b) in the following manner:

$$\lambda_s = {}^b\theta_s^m \mp |\theta_\Gamma|, \qquad (66)$$

145 The integral value of $\left[\frac{A'\times 31}{11323}\right]$ in (1).

146 See *Karaṇaratna* verses I.49(c,d)–50(a,b), Shukla (1979: 34–35).

147 This interpolation formula is not explicitly stated in the verse. The formula proposed by us here is a modification of (58), by changing the divisor to 600′.

where the correction is negative for the *ayana* in the northern hemisphere (first and second quadrants) and positive for the *ayana* in the southern hemisphere (third and fourth quadrants).[148]

12.1 EXPLANATION

The phenomenon of the vernal equinox (Γ) oscillating[149] about the *meṣādi* (M) is called trepidation and can be understood with the help of Figure 15. Figures 15a and 15b depict the instants when the vernal equinox (Γ) is positioned to the east and west of *meṣādi* (M), respectively. If $\widehat{MS_b} = {}^b\theta_s^m$, as obtained from (59), is the true *nirayana* longitude of the *bhujāntara* corrected Sun (S_b), and $\widehat{M\Gamma} = \theta_\Gamma$ is the position of the vernal equinox (Γ) with respect to *meṣādi* (M), then its corresponding *sāyana* longitude (λ_s) will be

$$\begin{aligned} \lambda_s = \widehat{\Gamma S_b} &= \widehat{MS_b} \mp \widehat{M\Gamma} \\ &= {}^b\theta_s^m \mp |\theta_\Gamma| \,. \end{aligned} \tag{67}$$

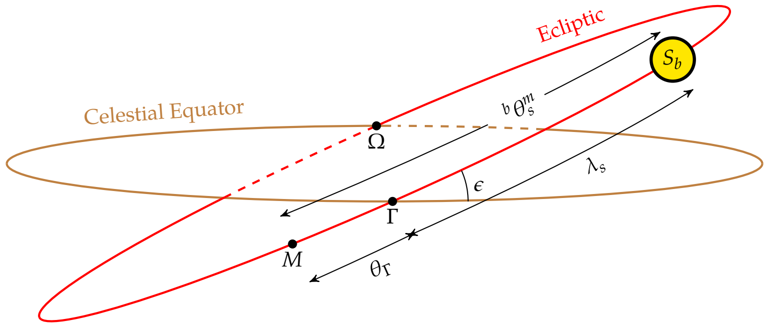
The position (θ_Γ) of the vernal equinox with respect to *meṣādi* (M) can be computed knowing the characteristics of the oscillation, i.e., amplitude, time period, and the position of the vernal equinox at some epoch. Knowing this model from the standard texts,[150] the amplitude ($[\theta_\Gamma]_{max}$) and the time period of the oscillation, considered in *Tithinirṇaya*, are taken to be 1440′ or 24° and 7380 *kali* years respectively. Further, at the instant of *kalyādi*, the position of the vernal equinox is considered to be $\theta_\Gamma = 0°$ and moving to the east of the *meṣādi* (M). If K_y is the number of *kali* years elapsed since the start of *kaliyuga*, then the number of oscillations completed by the vernal equinox (Γ) about the *meṣādi* (M) is given by the *ayana* as

$$ayana\ (\bar{A}) = \frac{K_y}{7380}\ \text{(osc)}. \tag{68}$$

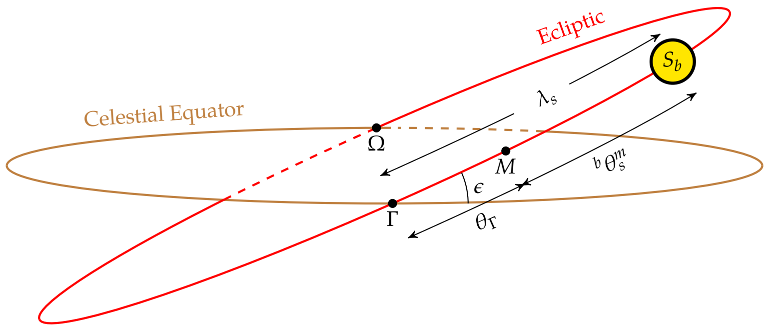
If K_y^e and K_y^{se} are the *kali* years elapsed till epoch (1610424) and since the epoch ($A' = A - 1610424$) respectively, then employing (4) and (7), we have

$$\begin{aligned} K_y &= K_y^e + K_y^{se} \\ &= \left[1610424 \times \frac{R_s}{D_c}\right] + \left[A' \times \frac{R_s}{D_c}\right] \\ &\approx 4409 + \left[A' \times \frac{31}{11323}\right]. \end{aligned} \tag{69}$$

148 Refer Footnote 130.
149 In Indian astronomy, there are two theories to describe the motion of the equinox. They are Trepidation and Precession. See Ramasubramanian and Sriram (2011: 14–15) for more details.
150 Refer footnote 144.



(a)



(b)

Figure 15: Diagrams depicting the oscillation of the vernal equinox (Γ) about *meṣādi* (M) and showing the instants when the vernal equinox (Γ) is to (a) the east of *meṣādi* (M) and (b) the west of *meṣādi* (M).

Thus, employing (69) in (68), we have

$$ayana\ (\bar{A}) = \left[\frac{4409 + \left[\frac{A' \times 31}{11323}\right]}{7380}\right] (\text{osc}) = \left[\frac{4409 + \left[\frac{A' \times 31}{11323}\right]}{615}\right] (\textit{rāśi}), \tag{70}$$

which is equivalent to (64).

The geometrical significance of *ayana* ($\bar{A}$) and the computation of the motion (θ_Γ) of the vernal equinox (Γ) with respect to *meṣādi* (M) from the *ayana* ($\bar{A}$) can be understood with the help of Figure 16. This figure is similar to Figure 15 and depicts the direction of motion of the Sun (S_b) on the ecliptic. The *sāyana* longitude ($\widehat{\Gamma S_b} = \lambda_s$) of the Sun ($S_b$) indicates its position on the ecliptic and

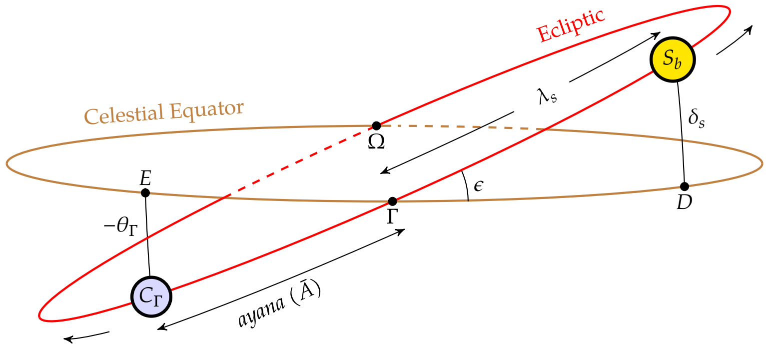


Figure 16: A diagram indicating the directions of motion of the Sun and the celestial body C_Γ on the ecliptic.

$\widehat{DS_b} = \delta_s$ is its corresponding declination given by[151]

$$\delta_s = \sin^{-1}(\sin\epsilon \times \sin\lambda_s)\,, \tag{71}$$

where $\epsilon = 24^\circ$ is the obliquity of the ecliptic. Thus, the declination of the Sun varies between $+24^\circ$ and -24°. In *Tithinirṇaya*, the amplitude of oscillation of the vernal equinox about *meṣādi* is also considered to be 24°. Perhaps due to this coincidence, the text proposes a formula for the computation of trepidation which is analogous to the declination formula of the Sun. In this analogy, the *ayana* corresponds to the longitude (λ_s) of the Sun, and the motion (θ_Γ) of the equinox corresponds to the declination (δ_s) of the Sun. This can be better understood with the help of the fictitious celestial body C_Γ (see Figure 16) moving on the ecliptic in the direction opposite to the motion of the Sun, with a revolution period equal to the time period of trepidation, i.e., 7380 *kali* years. The position of the celestial body C_Γ on the ecliptic is indicated by *ayana* ($\widehat{\Gamma C_\Gamma} = \bar{A}$), which is measured clockwise with respect to the vernal equinox (Γ). The declination ($\widehat{EC_\Gamma}$) of the celestial body (C_Γ) corresponds to the motion (θ_Γ) of the vernal equinox and computed from a relation analogous to (71) as follows:[152]

$$\theta_\Gamma = \sin^{-1}\left(\sin\epsilon \times \sin\bar{A}\right). \tag{72}$$

We have shown in Table 5 that the values of the motion (θ_Γ) of the vernal equinox computed from (72) are indeed close to the values given in the verses.

151 See *Tantrasaṅgraha* section II.11, Ramasubramanian and Sriram (2011: 78), for its derivation. Also, see *Āryabhaṭīya* verse 24 in the *Gola* chapter, Shukla and Sarma (1976: 132), *Laghubhāskarīya* verse II.16, Shukla (1963: 24–25), *Śiṣyadhīvṛddhidatantra* verse II.17, Chatterjee (1981: 39–41).

152 Refer footnote 144.

From Figure 16 it is evident that the sign of the declination of C_Γ is negative for the *ayana*s in first and second quadrants, measured clockwise from Γ, i.e., $0^\circ \leq ayana \leq 180^\circ$, and positive for the *ayana*s in third and fourth quadrants, i.e., $180^\circ \leq ayana \leq 360^\circ$. Hence, from (67), the true *sāyana* longitude (λ_s) of the *bhujāntara* corrected Sun (S_b) will be

$$\lambda_s = \begin{cases} {}^b\theta_s^m - |\theta_\Gamma| & , 0^\circ \leq ayana \leq 180^\circ \\ {}^b\theta_s^m + |\theta_\Gamma| & , 180^\circ \leq ayana \leq 360^\circ \end{cases}$$

which is equivalent to (66). It is worth noting that, as per this model, the rate of motion of the equinox,[153] in seconds per year, ranges from 0″, when *ayana* ($\bar{A}$) = $90^\circ, 270^\circ$, to 71.43″, when *ayana* ($\bar{A}$) = $0^\circ, 180^\circ$, with a mean rate observed to be approximately $(1440 \times 60 \times 4/7380) \approx 46.8''$/ year.

Further, at present, when 5125 *kali* years have elapsed, the *ayana* ($\bar{A}$) and motion (θ_Γ) of the equinox, from (68) and (72), are computed to be 249.99° and 22.47° respectively. As the current *ayana*, $\bar{A} = 249.99^\circ$, lies in the third quadrant, the *sāyana* longitude (λ_s) of the Sun is obtained by adding the motion ($\theta_\Gamma = 22.47^\circ$) of the equinox to the *nirayana* longitude (${}^b\theta_s^m$) of the Sun.

13 *UDAYĀNTARA* CORRECTION: ACCOUNTING THE OBLIQUITY OF THE ECLIPTIC

THE TITHINIRṆAYA does not discuss the *udayāntara* correction as a part of the sequence of corrections, but for the sake of completeness, we briefly discuss its purpose and procedure here. In modern astronomy, 'the equation of time' constitutes the time difference between the instants of true and mean sunrise.[154] In Indian astronomy, this time difference is accounted for by two distinct corrections: *bhujāntara* and *udayāntara*. We have already discussed the rationale of *bhujāntara* correction in Section 7. Now, we shall explain the second correction: *udayāntara*. The purpose of the *udayāntara* correction is to account for the obliquity of the ecliptic. The discussion until now has assumed zero obliquity of the ecliptic, i.e., the ecliptic coincides with the celestial equator as shown in Figures 3, 5, 7, 9, and 11. However, the ecliptic has an obliquity of $\epsilon = 24^\circ$. The *udayāntara* correction accounts for the time difference ($\Delta t^u = t^b \sim t^u$) between the instants of true sunrise at L' before and after considering the obliquity of the ecliptic, and can be understood with the help of Figure 17.

Figure 17 depicts the instant (t^b) of true sunrise at L', neglecting the obliquity of the ecliptic. Here, the true Sun (S_b), positioned at $\widehat{\Gamma S_b} = \Gamma \widehat{P_N} S_b = \lambda_s$ on the

153 The rate of motion of the equinox is computed taking the derivative of (72).

154 See Ramasubramanian and Sriram (2011: 82,464–465).

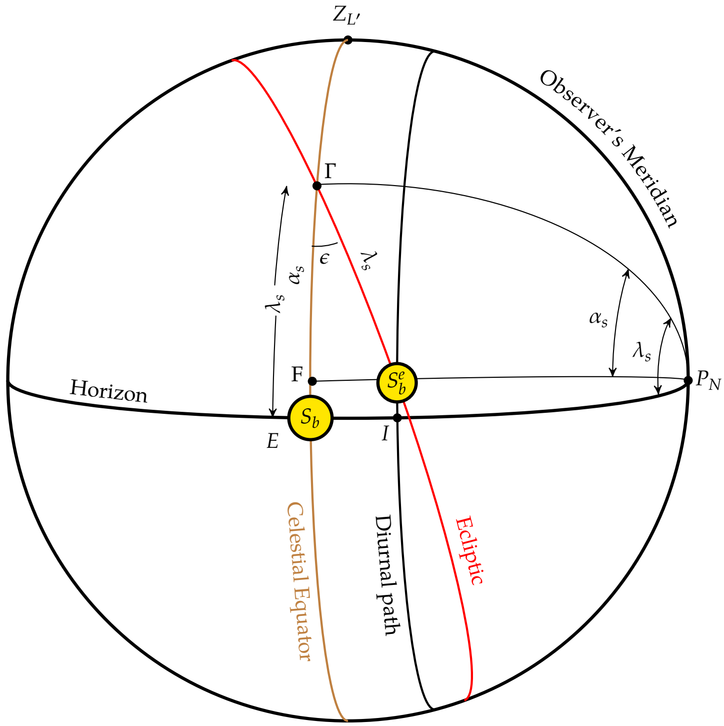


Figure 17: A diagram showing the instant (t^b) of true sunrise at L' and the effect of the obliquity of the ecliptic on the instant of true sunrise.

celestial equator, is just about to rise at the cardinal east (E). When the obliquity (ϵ) of the ecliptic is accounted for, the true Sun S_b^e will now be positioned on the ecliptic at $\widehat{\Gamma S_b^e} = \lambda_s$, which will not be on the horizon. In our figure, the Sun (S_b^e) has already risen. As we are interested in the instant (t^u) of true sunrise at L', one must travel back (or forward) in time in order to observe the Sun at the intersection (I) of the diurnal circle and the horizon. If F is the point of intersection of the arc of the meridian drawn through S_b^e and the celestial equator, the time taken for the true Sun to traverse from I to S_b^e on its diurnal path is equal to the time taken for a point on the celestial equator to traverse from E to F. We have

$$\begin{aligned} \widehat{EF} = E\widehat{P_N}F &= E\widehat{P_N}\Gamma - F\widehat{P_N}\Gamma \\ &= \widehat{\Gamma E} - \widehat{\Gamma F} \\ &= \lambda_s - \alpha_s, \end{aligned} \tag{73}$$

where α_s is the right ascension of the Sun corresponding to λ_s given by the expression[155]

$$\alpha_s = \sin^{-1}\left(\frac{\sin\lambda_s \times \cos\epsilon}{\cos\delta_s}\right). \qquad (74)$$

Here, $\epsilon = 24^\circ$ is the obliquity of the ecliptic, and δ_s is the declination of the Sun as given by (71). If Δt^u is the sidereal time taken for the diurnal motion of $\lambda_s - \alpha_s$, which is approximately the time difference $(t^u \sim t^b)$ between the instants of sunrise before and after considering the obliquity of the ecliptic, the angle traversed by the planet in this time interval is known as the *udayāntara* correction (Δ_p^u) of the planet. As the Sun (S_b^e) approximately traces a complete diurnal circle of 360° or $21600'$ in a sidereal day, the time taken by the Sun (S_b^e) to traverse the diurnal path by $\lambda_s - \alpha_s$ (min) will be

$$\Delta t^u = \frac{\lambda_s - \alpha_s}{21600'} \text{ (sidereal day)}. \qquad (75)$$

Thus, the *udayāntara* correction (Δ_p^u) of the planet — the angle (Δ_p^u), in minutes, traversed by the planet in the time interval Δt^u — will be

$$\Delta_p^u = \Delta t^u \times \dot{\theta}_p^t = \frac{\lambda_s - \alpha_s}{21600'} \times \dot{\theta}_p^t \text{ (min)}, \qquad (76)$$

where $\dot{\theta}_p^t$ is the true rate of motion of the planet in min/day.[156]

Hence, the *udayāntara* corrected planet (θ_p^u) will be

$$\theta_p^u = {}^b\theta_p^m \mp \left|\Delta_p^u\right|, \qquad (77)$$

where ${}^b\theta_p^m$ is the true longitude of the *bhujāntara* corrected planet.[157] The correction is negative for $(\lambda_s - \alpha_s) \geq 0$, which happens when λ_s is in the first and third quadrants, and positive for $(\lambda_s - \alpha_s) \leq 0$, which happens when λ_s is in second and fourth quadrants.[158] This correction was usually ignored by astronomers before the advent of Śrīpati (eleventh century CE).[159] Though the present work is composed after his period, it has not been considered in the text.

155 See Yelluru and Kolachana (2023: 171–173), Kolachana, Mahesh, and Ramasubramanian (2018: 3), and *Tantrasaṅgraha* section II.11, Ramasubramanian and Sriram (2011: 78), for its derivation.

156 Refer footnote 104.

157 In the case of the Sun and the Moon, ${}^b\theta_s^m$ and ${}^b\theta_m^m$ can be obtained from (59) and (60) respectively.

158 See Kolachana, Mahesh, and Ramasubramanian (2018: 12).

159 See Sastri (1957: XXXVI), Shukla (1963: 28), Shukla (1960: 114–115).

14 *CARADALA* CORRECTION: FOR AN OBSERVER'S LATITUDE OF 12.78°

अर्कपूज्यादिवाक्योक्तदोर्विलिप्ताश्चरोदिताः ।[160]
अर्कपूज्यः सुधाकरः रतिक्रीडो नुतः प्रभुः ॥ १९ ॥[161]
अलङ्कृष्णो हितोद्देशो गतिभूतः स्मरार्दितः ।[162]
शशिधातेति वाक्यानि ज्ञोनन्तोऽत्र तु हारकः ॥ २० ॥[163]
तदेकदिनगा लिप्ता ग्रहाणां स्वस्वभुक्तयः ।[164]
चरार्धात् स्वस्वभुक्तिघ्नादनन्ताङ्गहृताः कलाः ।[165]
ऋणं प्रातर्धनं सायमुत्तरे दक्षिणेऽन्यथा ॥ २१ ॥ ॥ अनुष्टुभ् ॥
देशान्तरदोर्विवरजसंस्कारविधिर्विधीयते मध्ये ।[166]
चरदलसंस्कारविधिः स्फुटक्रियानन्तरं सद्भिः ॥ २२ ॥[167] ॥ आर्या ॥

arkapūjyādivākyoktadorviliptāścaroditāḥ |
arkapūjyaḥ sudhākaraḥ ratikrīḍo nutaḥ prabhuḥ || 19 ||
alaṅkṛṣṇo hitoddeśo gatibhūtaḥ smarārditaḥ |
śaśidhāteti vākyāni jñonanto'tra tu hārakaḥ || 20 ||
tadekadinagā liptā grahāṇāṃ svasvabhuktayaḥ |
carārdhāt svasvabhuktighnādanantāṅgahṛtāḥ kalāḥ |
ṛṇaṃ prātardhanaṃ sāyamuttare dakṣiṇe'nyathā || 21 || || *anuṣṭubh* ||
deśāntaradorvivarajasaṃskāravidhirvidhīyate madhye |
caradalasaṃskāravidhiḥ sphuṭakriyānantaraṃ sadbhiḥ || 22 || || *āryā* ||

The Rsine in *vilipti*s mentioned in *vākyas* beginning with *arkapūjya* (1110) etc., are called *cara*s. The *vākyas* are thus: *arkapūjya* (1110), *sudhākara* (2197), *ratikrīḍa* (3262), *nutaprabhu* (4260), *alaṅkṛṣṇa* (5130), *hitoddeśa* (5868), *gatibhūta* (6463), *smarārdita* (6825), *śaśidhāta* (6955). Here, *jño'nanta* (600) is the divisor [for interpolation]. The respective rates of motion of the planets [is equal to] the minutes (*lipti*s) traversed by them in one day. The minutes (*kalā*s) are [the result obtained] from half the *cara* multiplied by their respective

160 Bhikṣu (n.d.) has the reading अर्कपूज्याप्तवाक्याप्त।

161 Bannañje (1974*b*: 189) notes अर्कः पूज्यः as an alternate reading. Bhikṣu (n.d.) has the reading अर्कपूज्या (1110) सदाकारो (2187) रतिकडा (3162) नतुभूपम् (4260)।

162 Bannañje (1974*b*: 189) notes स्मरादितः as an alternate reading and स्मराजितः as a scribal error. Bhikṣu (n.d.) has the reading स्मराजितः।

163 Bhikṣu (n.d.) has the reading ज्ञोनन्त-स्तत्र हारकः। A half verse देशान्तरं रवेर्बाहुः भागघ्नन्तद्धरैरपि is also found after verse 20.

164 Bannañje (1974*b*: 190) notes an alternate reading स्वस्वभूतयः as a scribal error. Bhikṣu (n.d.) has the reading तदेकदिनभागालिप्ताः।

165 Bannañje (1974*b*: 190) notes an alternate reading स्वस्वभूतिघ्नात् as a scribal error. Bhikṣu (n.d.) has the reading चरार्धं स्वस्वभुक्त्या ज्ञानताङ्गहृताः कलाः ।

166 Bannañje (1974*b*) notes देशान्तरे दोर्विवरसंस्कार as an alternate reading.

167 This verse is missing in Bhikṣu (n.d.).

> rates of motion and divided by *anantāṅga* (3600). [The result is] negative (i.e., subtracted) for the morning, [and] positive (i.e., added) for the evening [if the Sun is] in the northern [hemisphere], [and] otherwise [if the Sun is] in the southern [hemisphere]. The procedure of *deśāntara* correction and the *bhujāntara* correction (*dorvivaraja*) is recommended in [the computation of] the mean [planets], and the procedure of *caradala* correction is recommended [to be applied] after determining the true [planets], by the learned.

The above verses prescribe the *caradala* correction (Δ_p^{ca}) for the planet (p) to obtain its true longitude (θ_p^t) at the instant (t^{ca}) of true sunrise for an observer Q understood to be at a latitude (ϕ) of 12.78°.[168] This correction accounts for the time difference ($\Delta t^{ca} = t^{ca} \sim t^u$)[169] between the instants of true sunrise at Q ($\phi = 12.78°$) and L' ($\phi = 0°$). Verses 19 and 20, through the phrases *arkapūjya*, etc., state the values of twice the ascensional difference, or *cara* ($2\Delta\alpha$), in *gurvakṣaras*,[170] for an observer situated at a latitude (ϕ) of 12.78°, at every 600′ interval of the *sāyana* longitude of the Sun (λ_s), as summarized in Table 6.

To determine the *cara* ($2\Delta\alpha$) corresponding to the *sāyana* longitude (λ_s) of the Sun which lies within any given interval, the verses hint at an interpolation formula, as was also previously observed in verses 17 and 18. If $(2\Delta\alpha)_i$ and $(2\Delta\alpha)_{i+1}$ are the *cara* values corresponding to the successive values of *sāyana* longitude of the Sun, $(\lambda_s)_i$ and $(\lambda_s)_{i+1}$ respectively, then the desired *cara* ($2\Delta\alpha$) corresponding to $(\lambda_s)_j$ which lies in between $(\lambda_s)_i$ and $(\lambda_s)_{i+1}$ can be obtained by the following interpolation:[171]

$$2\Delta\alpha = (2\Delta\alpha)_i + \frac{(2\Delta\alpha)_{i+1} - (2\Delta\alpha)_i}{600'} \times \left((\lambda_s)_j - (\lambda_s)_i\right). \tag{78}$$

Verses 21 and 22 prescribe the procedure for applying the *caradala* correction (Δ_p^{ca}) to a planet (p). This correction is applied to the true *nirayana* longitude (${}^b\theta_p^m$) of the *bhujāntara* corrected planet (p) at true sunrise at L'.[172] The following rule is prescribed in the above verses

168 See Section 14.1.2 for more details.

169 If the *udayāntara* correction is neglected, as in the *Tithinirṇaya*, then t^u may be approximated to t^b.

170 Though the units of the *cara* are stated to be *viliptis* in the verse, Bannañje (1974*b*: 190) correctly notes that the units should be in *gurvakṣaras*. It may be noted that 1 *ghaṭikā* = 60 *vighaṭikās* = 3600 *gurvakṣaras*.

171 Refer footnote 147.

172 This correction must be actually applied to the true *nirayana* longitude (θ_p^u) of the *udayāntara* corrected planet, as obtained from (77). As the *udayāntara* correction is ignored in the sequence of corrections in *Tithinirṇaya*, this correction is applied to the true *nirayana* longitude (${}^b\theta_p^m$) of the *bhujāntara* corrected planet, as obtained from (59) and (60) for the Sun and the Moon respectively.

S. No.	λ_s	δ_s	$\sin(\Delta\alpha)$	*Cara* $(2\Delta\alpha)$			
				calculated	in *Tithinirṇaya*		
	min	min		*vighaṭikā*	phrase	*gurvakṣara*	*vighaṭikā*
1	600	243.01	0.0161	18.41	*arkapūjya*	1110	18.5
2	1200	479.79	0.0319	36.52	*sudhākara*	2197	36.62
3	1800	704.04	0.0471	54.01	*ratikrīḍa*	3262	54.37
4	2400	909.35	0.0614	70.45	*nutaprabhu*	4260	71.00
5	3000	1089.26	0.0744	85.31	*alaṅkṛṣṇa*	5130	85.5
6	3600	1237.48	0.0854	97.95	*hitoddeśa*	5868	97.8
7	4200	1348.23	0.0938	107.67	*gatibhūta*	6463	107.72
8	4800	1416.78	0.0992	113.82	*smarārdita*	6825	113.75
9	5400	1440.00	0.1010	115.92	*śaśidhāta*	6955	115.92

Table 6: The values of *cara* $(2\Delta\alpha)$ for the corresponding values of the *sāyana* longitude of the Sun (λ_s).

$$\begin{aligned}\theta_p^t &= {}^b\theta_p^m \mp \left|\Delta_p^{ca}\right| \\ &= {}^b\theta_p^m \mp \left|\frac{cara}{2} \times \frac{svasvabhukti}{anant\bar{a}\dot{n}ga}\right| (kal\bar{a}s) \\ &= {}^b\theta_p^m \mp \left|\frac{2\Delta\alpha}{2} \times \frac{\dot{\theta}_p^t}{3600}\right| (\text{min}), \end{aligned} \tag{79}$$

where $\dot{\theta}_p^t$ is the true motion of the planet in min/day and $2\Delta\alpha$ is the *cara* in *vighaṭikās*. The correction is subtracted for sunrise and added for sunset if the *sāyana* Sun is in the northern hemisphere (first and second quadrants). The correction is done otherwise if the *sāyana* Sun is in the southern hemisphere (third and fourth quadrants).[173] The verses also prescribe that the *cara* correction is done only after obtaining the true planet, whereas *deśāntara* and *bhujāntara* corrections are applied on the mean planet.

14.1 EXPLANATION

14.1.1 *Significance of cara*

The *cara*, or twice the ascensional difference $(2\Delta\alpha)$, is the time increment or decrement in the length of the day for a non-equatorial observer at Q (see Figure 2)

173 Refer Footnote 130.

with respect to an equatorial observer at L'. This time difference arises due to the differences in the instants of sunrise and sunset at different latitudes on the earth. The rationale for the correction can be understood with the help of Figure 18. Figure 18a is similar to Figure 17 and depicts an *udayāntara* corrected Sun (S_u) at the instant (t^u) of true sunrise for an observer at L'. This observer (L') on the equator views the Sun (S_u) rising and setting at I and J respectively on the 6 o'clock circle.[174] The length of the day (sunrise (X) to sunset (Y)) is the time taken for the Sun (S_u) to traverse from I to J along the diurnal path, whose magnitude is given by the angular measure $I\widehat{P_N}J = X\widehat{P_N}Y = 180^\circ$. Figure 18b also depicts the same instant (t^u) as Figure 18a,[175] but for an observer Q at a northern latitude ($\widehat{NP_N} = \phi$). This observer (Q) views the Sun (S_u) rising and setting at X and Y, respectively, at the horizon. The length of the day, in this case, is the time taken for the Sun (S_u) to traverse from X to Y along the diurnal path, which is indicated by an angular measure $X\widehat{P_N}Y$ given by

$$\begin{aligned} X\widehat{P_N}Y &= X\widehat{P_N}I + I\widehat{P_N}J + J\widehat{P_N}Y \\ &= 180^\circ + 2\Delta\alpha, \end{aligned} \tag{80}$$

where $X\widehat{P_N}I = J\widehat{P_N}Y = \Delta\alpha$ and $I\widehat{P_N}J = 180^\circ$.

As the time difference (Δt^{ca}) between the instants of sunrise (or sunset) for the observers at Q (ϕ) and at L' ($\phi = 0^\circ$) is the time required to cover the diurnal path $X\widehat{P_N}I$ (or $J\widehat{P_N}Y$) $= \Delta\alpha$, the total time increment or decrement in the length of the day, in *vighaṭikā*s,[176] is given by[177]

$$\begin{aligned} \textit{cara} = 2\Delta\alpha &= 2 \times \sin^{-1}(\tan\phi \times \tan\delta_s) \text{ (degrees)} \\ &= 2 \times \sin^{-1}(\tan\phi \times \tan\delta_s) \times \frac{3600}{360^\circ} \text{ (\textit{vighaṭikās})}, \end{aligned} \tag{81}$$

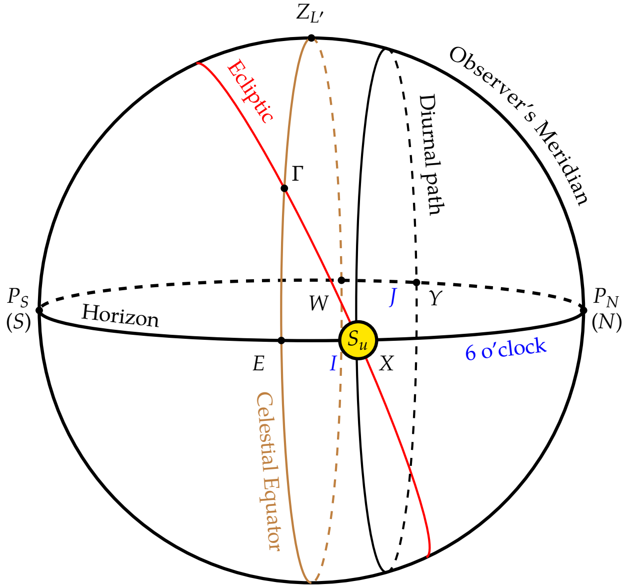
where δ_s is the declination of the Sun as given by (71).

174 The 6 o'clock circle is the great circle passing through the cardinal east (E) and west (W), and the celestial poles (P_N or P_S) and bisects the diurnal path of the sun. For an equatorial observer, the 6 o'clock circle coincides with the horizon.

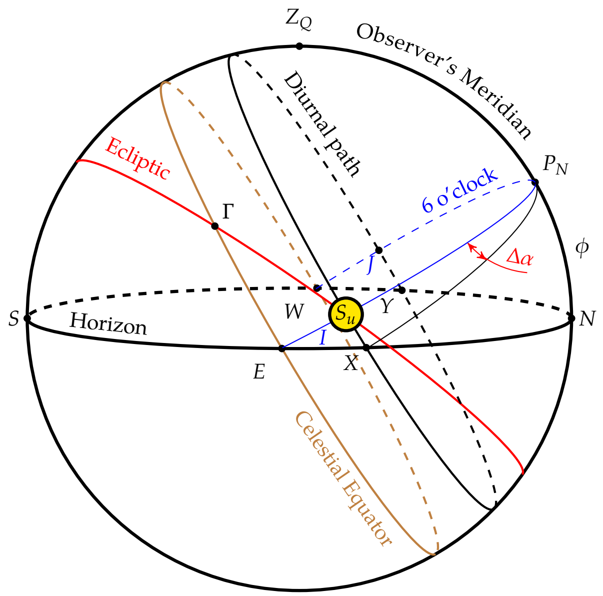
175 See the position of the Sun (S_u) at 'I' on the 6 o'clock circle.

176 Approximating 1 sidereal day ≈ 1 mean civil day = 3600 *vighaṭikā*s.

177 See Kolachana, Mahesh, Montelle, et al. (2018) for its derivation. Also, see *Āryabhaṭīya* verse 26 in the *Gola* chapter, Shukla and Sarma (1976: 135–136), *Laghubhāskarīya* verses II.17–18, Shukla (1963: 25–26), *Mahābhāskarīya* verses III.6–7, Shukla (1960: 62–65), *Śiṣyadhīvṛddhidatantra* verse II.18, Chatterjee (1981: 39–43), *Tantrasaṅgraha* section II.11, Ramasubramanian and Sriram (2011: 76–80), *Karaṇapaddhati* verse VIII.15–18, Pai, Ramasubramanian, et al. (2018: 252–256).



(a)



(b)

Figure 18: Diagrams showing the diurnal path of the Sun (S_u) for the observers having the same meridian but (a) one at L' on the equator ($\phi = 0^\circ$) and (b) the other at Q on any northern latitude (ϕ).

14.1.2 Latitude for which cara is computed in Tithinirṇaya

The computed values of *cara* ($2\Delta\alpha$), using (81), approximately match with the values mentioned in the verses for an observer at latitude (ϕ) of $12.78^{\circ}N$, as shown in Table 6. This implies that the author wishes to compute the *tithi* for this latitude. Kāvu ($\phi = 12.53^{\circ}N$), the hometown of Śrī Trivikramapaṇḍitācārya (suggested author of this *Tithinirṇaya*), near Kāsargoḍ, Kerala, is situated near this latitude.[178] Udupi ($\phi = 13.34^{\circ}N$), where the Mādhva community is concentrated, is also situated close to this latitude, and thus this text could have been intended for the computations there as well.

14.1.3 Rationale for the caradala correction

The procedure for application of *cara*, as given in (79), can be understood as follows. As Figure 18b depicts the instant (t^u) of true sunrise at L', to determine the instant (t^{ca}) of true sunrise at Q, one should travel back (or forward) in time in order to observe the true Sun at the intersection (X) of the diurnal path of the Sun and the horizon. If $\Delta t^{ca} = \Delta\alpha$ (in *vighaṭikās*), computed using (81), is the sidereal time taken for the diurnal motion ($\widehat{XS_u}$) of the Sun, which is approximately the time difference ($t^{ca} \sim t^u$) between the instants of true sunrise for the observers at Q (ϕ) and at L' ($\phi = 0^{\circ}$), then the angle traversed by the planet in this time interval is known as the *caradala* correction (Δ_p^{ca}) of the planet (p) and is given by[179]

$$\Delta_p^{ca} = \Delta\alpha \times \frac{\dot{\theta}_p^t}{3600} \text{ (min)}, \tag{82}$$

where $\dot{\theta}_p^t$ is the true motion of the planet in min/day.[180] Hence, the *caradala* corrected planet (θ_p^t) is obtained by applying (82) to the *udayāntara* corrected planet (θ_p^u) as follows:

$$\begin{aligned} \theta_p^t = \theta_p^u \mp \left|\Delta_p^{ca}\right| &\approx {}^b\theta_p^m \mp \left|\Delta_p^{ca}\right| \\ &\approx {}^b\theta_p^m \mp \left|\Delta\alpha \times \frac{\dot{\theta}_p^t}{3600}\right|, \end{aligned} \tag{83}$$

which is equivalent to (79), because the *udayāntara* correction is neglected in *Tithinirṇaya*.

14.1.4 Sign of the caradala correction

The sign of the *caradala* correction is based on whether the diurnal motion of the Sun is considered forward or back in time to observe the true Sun at the

178 Refer footnote 30.

179 See *Karaṇaratna* verse I.39, Shukla (1979: 29), *Khaṇḍakhādyaka* verse I.22, Sengupta (1934).

180 Refer footnote 176.

horizon. When the diurnal motion of the Sun is considered forward or back in time, the *caradala* correction should be added or subtracted, respectively. This can be understood with the help of Figure 19, which is similar to Figure 18b, and depicts the diurnal path of the Sun when its true *sāyana* longitude (λ_s) falls in four different quadrants at the instant (t^u) of true sunrise at L' for an observer at Q. It is observed from Figures 19a and 19b that, when the true *sāyana* longitude (λ_s) of the *udayāntara* corrected Sun (S_u with declination δ_s) is in the first and second quadrants respectively, i.e., $0^\circ \leq \lambda_s \leq 180^\circ$, the true sunrise at Q happens $\Delta\alpha$ *vighaṭikā*s before the true sunrise at L', because the Sun, during its diurnal motion, reaches the horizon (X) before the 6 o'clock circle (I), and thus the *caradala* correction should be subtracted.

Similarly, it is observed from Figures 19c and 19d that, when the *sāyana* Sun (S_u with declination $-\delta_s$) is in the third and fourth quadrants respectively, i.e., $180^\circ \leq \lambda_s \leq 360^\circ$, the true sunrise at Q happens $\Delta\alpha$ *vighaṭikā*s after the true sunrise at L', because the Sun, during its diurnal motion, reaches the 6 o'clock circle (I) before reaching the horizon (X), and thus the *caradala* correction should be added.

14.1.5 *Caradala correction for the Sun and Moon*

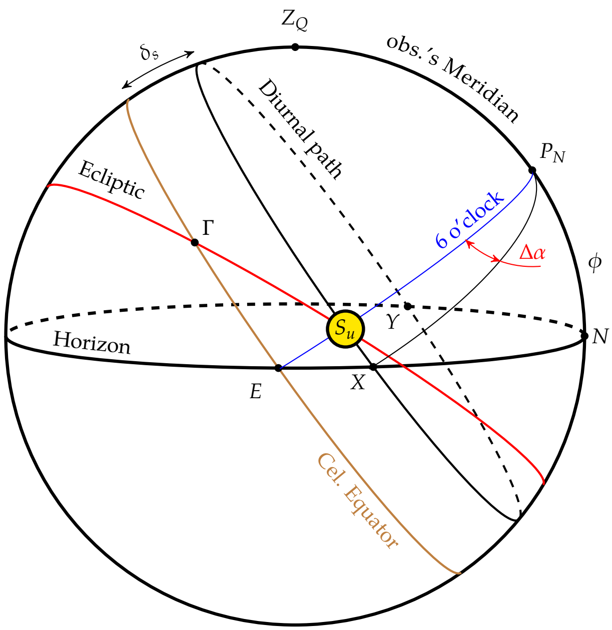
Substituting the values for the Sun and the Moon in (83), the true longitudes (θ_s^t and θ_m^t) of the Sun and the Moon at the instant (t^{ca}) of true sunrise for an observer at Q ($\phi = 12.78^\circ$), respectively, will be[181]

$$\theta_s^t = \theta_s^u \mp |\Delta_s^{ca}| \approx {}^b\theta_s^m \mp |\Delta_s^{ca}| = {}^b\theta_s^m \mp \left|\Delta\alpha \times \frac{\dot{\theta}_s^t}{3600}\right| \quad (84)$$

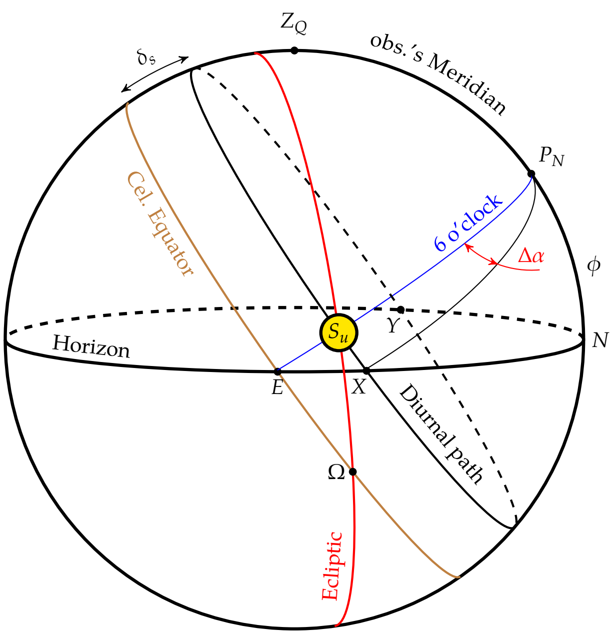
$$\theta_m^t = \theta_m^u \mp |\Delta_m^{ca}| \approx {}^b\theta_m^m \mp |\Delta_m^{ca}| = {}^b\theta_m^m \mp \left|\Delta\alpha \times \frac{\dot{\theta}_m^t}{3600}\right|, \quad (85)$$

where ${}^b\theta_s^m$ and ${}^b\theta_m^m$ are the true longitudes of the *bhujāntara* corrected Sun and Moon, as obtained from (59) and (60), respectively, and $\dot{\theta}_s^t$ and $\dot{\theta}_m^t$ are the true rates of motion of the Sun and Moon, respectively, and their computation will be discussed in the following section.

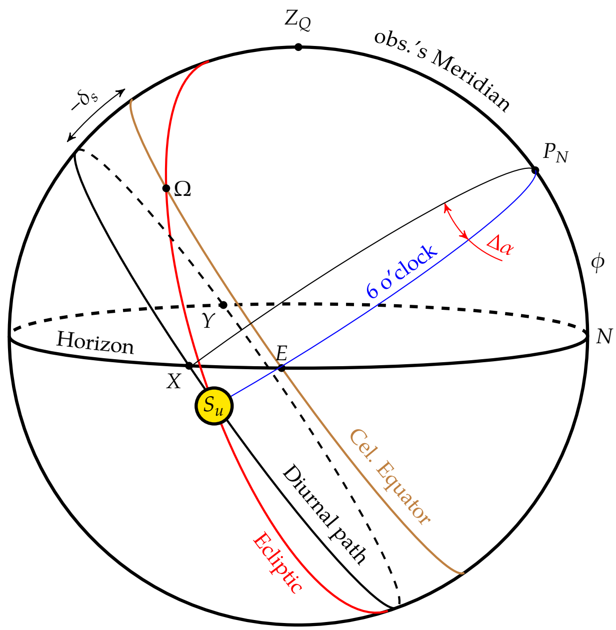
181 As the *udayāntara* correction is ignored in the *Tithinirṇaya*, $\theta_s^u \approx {}^b\theta_s^m$ and $\theta_m^u \approx {}^b\theta_m^m$.


Z_Q
δs
obs.'s Meridian
Diurnal path
P_N
Ecliptic
Γ
6 o'clock
Δα
φ
Υ
S_u
Horizon
N
E
X
Cel. Equator

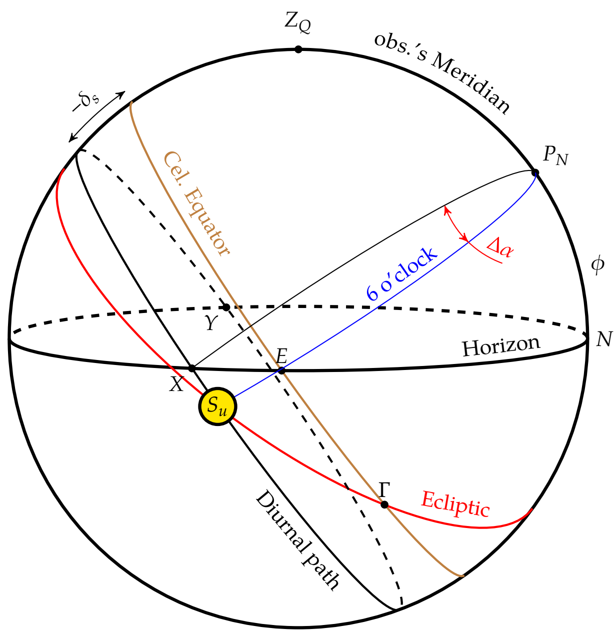
(a)


Z_Q
δs
obs.'s Meridian
P_N
Cel. Equator
6 o'clock
Δα
φ
Υ
S_u
Horizon
N
E
X
Diurnal path
Ω
Ecliptic

(b)



(c)



(d)

Figure 19: Diagrams showing the instant (t^{ca}) of sunrise at Q is, $\Delta\alpha$ *vighaṭikās* before the instant (t^u) of sunrise at the equator (L'), when the Sun (S_u) is in (a) quadrant-I (b) quadrant-II, and $\Delta\alpha$ *vighaṭikās* after the instant (t^u) of sunrise at the equator (L'), when the Sun (S_u) is in (c) quadrant-III (d) quadrant-IV.

14.1.6 True rate of motion of the planet

Even though (50) and (76) previously introduced the true rate of motion ($\dot{\theta}_p$) of the planet, this concept is addressed here for a purpose. The earlier corrections in *Tithinirṇaya*, such as *deśāntara* and *bhujāntara*,[182] utilize only the mean rate of motion of the planet. This is because the correction is performed with respect to the observer positioned at the center of the planet's orbit.[183] As the mean rates of motion ($\dot{\theta}_s^\circ$ and $\dot{\theta}_m^c$) of the Sun and the Moon, given by (4) and (16) respectively, are constant, their respective corrections of *deśāntara*, given by (36) and (37), and *bhujāntara*, given by (44) and (45), do not explicitly utilize the rates of motion, which are however implicit in the choice of multiplier and divisor.

The *caradala* correction is the only correction, in the *Tithinirṇaya*, which utilizes the true rate of motion ($\dot{\theta}_p^t$) of the planet. This is evident from the use of the phrase '*svasvabhukti*' in verse 21. The true rate of motion of the planet is the rate of angular displacement of the planet with respect to the observer at (O') in Figure 11. The procedure to obtain the true rates of motion ($\dot{\theta}_s^t$ and $\dot{\theta}_m^t$) of the Sun and the Moon is not discussed in the text *Tithinirṇaya*. Pai and Sriram (2023) give a detailed overview of the computations of true rates of motion in different astronomical texts. Bannañje (1974*b*: 190) and Vyāsadāsa (2007: 37) compute it in the following manner. If $({}^b\theta_p^m)_A$ and $({}^b\theta_p^m)_{A+1}$ in minutes are the true longitudes of the *bhujāntara* corrected planet at the instant (t^b) of true sunrise at L' on the *kali-ahargaṇa*s A and $A+1$ respectively, then the true motion of the planet $\dot{\theta}_p^t$ in min/day is given by[184]

$$\dot{\theta}_p^t = ({}^b\theta_p^m)_{A+1} - ({}^b\theta_p^m)_A. \tag{86}$$

Bhikṣu (n.d.), in his commentary on *Tithinirṇaya*, proposes the following expressions for the true rates of motion ($\dot{\theta}_s^t$ and $\dot{\theta}_m^t$) of the Sun and Moon, respectively:[185]

$$\dot{\theta}_s^t = \dot{\theta}_s^\circ \mp \dot{\theta}_s^\circ \left(\frac{3}{80} \times \frac{\text{Rsine difference}}{225'}\right) \tag{87}$$

$$\dot{\theta}_m^t = \dot{\theta}_m^c \mp \dot{\theta}_m^c \left(\frac{7}{80} \times \frac{\text{Rsine difference}}{225'}\right), \tag{88}$$

where $\dot{\theta}_s^\circ$, and $\dot{\theta}_m^c$ are the mean rates of motion of the Sun and the Moon, as obtained from (4) and (16) respectively.

182 See (40), (52) and (55).
183 See Section 7.1 for our discussion on 'mean' and 'true' parameters.
184 See *Laghubhāskarīya* verse II.15(c,d), Shukla (1963: 24), *Mahābhāskarīya* verse IV.18, Shukla (1960: 122).
185 See *Śiṣyadhīvṛddhidatantra* Appendix XIII, Chatterjee (1981: 318–319) for the derivation. Also see, *Laghubhāskarīya* verses II.9–13, Shukla (1963: 20–23), *Mahābhāskarīya* verses IV.14–17, Shukla (1960: 120–122), *Karaṇaratna* verses I.31(c,d)–32, Shukla (1979: 22–23).

15 ELAPSED *TITHI* AND THE ELAPSED TIME IN THE CURRENT *TITHI*

चन्द्रात् पूर्वोक्तसंस्कारकृताद् दिनपतिं त्यजेत् ।
शेषं षष्ट्या घटीभिस्तु विभज्याऽप्ता गता तिथिः ॥ २३ ॥ [186]
वर्तमानां द्वादशभिर्लिप्तिकाभिर्हरेत् तिथिम् ।
गतास्तु नाडिका ज्ञेयाः शिष्टा लिप्ताः प्रकीर्तिताः ॥ २४ ॥ ॥ अनुष्टुभ् ॥

candrāt pūrvoktasaṃskārakṛtād dinapatiṃ tyajet |
śeṣaṃ ṣaṣṭyā ghaṭībhistu vibhajyā'ptā gatā tithiḥ || 23 ||
vartamānāṃ dvādaśabhirliptikābhirharet tithim |
gatāstu nāḍikā jñeyāḥ śiṣṭā liptāḥ prakīrtitāḥ || 24 || || *anuṣṭubh* ||

From [the true longitude of] the Moon for which the earlier mentioned corrections are done, [the true longitude of] the Sun should be subtracted. After dividing the remainder by 60 *ghaṭikā*s the elapsed *tithi* is indeed obtained. One shall divide [the elapsed minutes of] the current *tithi* by 12 *lipti*s [from which] the elapsed [time in the current *tithi* in] *nāḍikā*s are to be known. The remainder are stated to be the [elapsed] minutes (*lipti*s).

The above verses prescribe the procedure to obtain the number of elapsed *tithi*s (E_{tithi}), and the elapsed time in *nāḍikā*s ($E_{ghaṭikā}$) and *lipti*s (E_{lipti}) in the current *tithi*. As described in the above verses, the number of *tithi*s elapsed (E_{tithi}) is obtained from the quotient of

$$\frac{(\theta_m^t - \theta_s^t)}{60(\textit{ghaṭikās})}, \tag{89}$$

where $(\theta_m^t - \theta_s^t)$ is the difference in the true longitudes of the Moon and the Sun in minutes. Normally, the denominator in (89) is considered to be 720′ or 12° in astronomical texts.[187] Presuming that the author, by the use of 60 *ghaṭikā*s as the denominator in (89), intended to indicate the average duration of a *tithi*, corresponding to an increase of 720′ in the longitudinal separation of the Moon and the Sun, (89) can be written as:[188]

$$\frac{(\theta_m^t - \theta_s^t)}{720'} = E_{tithi} + \frac{r_t}{720'}, \tag{90}$$

where the quotient E_{tithi} gives the number of *tithi*s elapsed and the remainder r_t gives the number of arc minutes (*lipti*s) elapsed in the current *tithi*.

186 Bhikṣu (n.d.) has the reading शिष्टाः लवाः श्रिया (12°) भक्ता लब्धास्तु तिथयो गताः।

187 See S. B. Rao (2000: 64–66), Ramasubramanian and Sriram (2011: 116–118).

188 This has vexed earlier commentators too. See Bannañje (1974*b*: 192).

Further, the verse prescribes the procedure to find the time elapsed (in *ghaṭikā*s) in the current *tithi* as follows:

$$\frac{r_t}{12(liptis)} = gatan\bar{a}\d{d}ik\bar{a}\text{s} + \frac{gataliptis}{12}$$
$$= E_{gha\d{t}ik\bar{a}} + \frac{E_{lipti}}{12}, \quad (91)$$

where the quotient $E_{gha\d{t}ik\bar{a}}$ (*gatanāḍikā*s) and the remainder E_{lipti} (*gatalipti*s) give the *nāḍikā*s and the *lipti*s elapsed in the current *tithi* respectively.

15.1 EXPLANATION

Tithi is a time duration in which the Moon increases the lead over the Sun by 12° or 720′. As the maximum lead that the Moon can have over the Sun is 360°, there can be a total of thirty *tithi*s (360 ÷ 12). If θ_s^t and θ_m^t are the true longitudes (in arc minutes) of the Sun and the Moon respectively at the instant (t^{ca}) of true sunrise at Q, then the number of *tithi*s elapsed is naturally given by[189]

$$\frac{\theta_m^t - \theta_s^t}{720'} = E_{tithi} + \frac{r_t}{720'}, \quad (92)$$

where E_{tithi} is the integral number of *tithi*s elapsed and r_t (arc minutes) is the elapsed portion of the current *tithi* before sunrise. If $\dot{\theta}_s^t$ and $\dot{\theta}_m^t$, in min/day, are the true rates of motion of the Sun and the Moon respectively, then the time elapsed in the current *tithi*, in *ghaṭikā*s, before sunrise will be given by

$$= \frac{r_t}{\left(\dot{\theta}_m^t - \dot{\theta}_s^t\right)} \times 60\ (gha\d{t}ik\bar{a}s). \quad (93)$$

Here, the author approximates $(\dot{\theta}_m^t - \dot{\theta}_s^t)$ to 720′/day. Hence, (93) will be reduced to

$$\frac{r_t \times 60}{720} = \frac{r_t}{12} = E_{gha\d{t}ik\bar{a}} + \frac{r_g}{12}, \quad (94)$$

where the integral part $E_{gha\d{t}ik\bar{a}}$ gives the *ghaṭikās* elapsed in the current *tithi* before sunrise. Multiplying the fractional part of the elapsed *ghaṭikā*s by 60 gives the additional *vighaṭikās* elapsed in the current *tithi*:

$$\frac{r_g \times 60}{12}(vigha\d{t}ik\bar{a}s) = r_g \times 5\ (vigha\d{t}ik\bar{a}s).$$

189 See *Laghubhāskarīya* verses II.26(c,d)–27, Shukla (1963: 31), *Mahābhāskarīya* verses IV.31–32, Shukla (1960: 130), *Khaṇḍakhādyaka* verse I.25, Sengupta (1934), *Karaṇaratna* verses I.41–42(a,b), Shukla (1979: 29–30), *Śiṣyadhīvṛddhidatantra* verse II.22, Chatterjee (1981: 43), *Laghumānasa* verse IV.4, Shukla (1990: 142), *Tantrasaṅgraha* verses II.55–59, Ramasubramanian and Sriram (2011: 116–118).

However, as five *vighaṭikās* correspond to one *lipti*,[190] an additional r_g *liptis* have elapsed in the current *tithi*. Thus, denoting r_g *liptis* as E_{lipti}, (94) can be rewritten as:

$$\frac{r_t}{12} = E_{ghaṭikā} + \frac{E_{lipti}}{12}, \qquad (95)$$

which is equivalent to (91).

16 DETERMINING *VIDDHAIKĀDAŚĪ*

एकाऽतिद्वादशीवृद्धौ नो चेद् वृद्धौ तु षोडश ।
द्व्येकलिप्ती समे ह्रासे चतुष्कादुत्तरं त्विदम् ॥ २५ ॥ ॥ अनुष्टुभ् ॥

ekā'tidvādaśīvṛddhau no ced vṛddhau tu ṣoḍaśa |
dvyekaliptī same hrāse catuṣkāduttaraṃ tvidam || 25 || || anuṣṭubh ||

In the case of *vṛddhi* not being beyond twelve [*liptis*] (i.e., *samavṛddhi*), one [*nāḍikā* has to be checked], but in the case of [*ati*]*vṛddhi*, sixteen [*liptis* have to checked]. In *sama* and *hrāsa*, two and one *lipti* [respectively are to be checked]. This [rule] is also in addition to the quartet [of *nāḍikās*] before [sunrise which are to be checked for the presence of *daśamī* in order to postpone the *ekādaśī* fast].

The above verse, attributed to Śrī Trivikramapaṇḍitācārya,[191] tersely prescribes the rules for postponing the *ekādaśī* fast based on the type of *daśamī-tithi*. It gives the time interval to be checked before the *aruṇodayakāla*[192] for the presence of *daśamī-tithi*. The types of the *daśamī-tithis* and the time interval before *aruṇodayakāla* which will lead to the postponement of the *ekādaśī* fast are summarized in Table 7.

190 Considering the average duration of *tithi* (12° or 720′ increase in the lead of the Moon with respect to the Sun) ≈ 60 *ghaṭikās*, then 720 *liptis* = 60 *ghaṭikās* $\implies$ 12 *liptis* = 1 *ghaṭikā* $\implies$ 1 *lipti* = 5 *vighaṭikās*.

191 See *Ekādaśī-nirṇaya* verse 30, B. P. N. Rao (1994: 34), *Smṛtimuktāvalī*, Giri Ācārya (2016: 147–148), *Karmasiddhānta*, Rāmanāthācārya (2013: 93).

192 *Aruṇodayakāla* is the time duration of four *ghaṭikās* before sunrise.

Type of daśamī-tithi	*Time before aruṇodayakāla*
hrāsa	1 *lipti*
sama	2 *lipti*s
samavṛddhi	1 *ghaṭikā*
ativṛddhi	16 *lipti*s

Table 7: Time before *aruṇodayakāla* to be checked for the presence of *daśamī*.

16.1 EXPLANATION

The *Ekādaśī-nirṇaya* of Śrī Vādirājatīrtha provides a lucid explanation of this verse,[193] which is illustrated using Figure 20. Figure 20a depicts the *tithi* transitions *daśamī*-*ekādaśī*, *ekādaśī*-*dvādaśī*, and *dvādaśī*-*trayodaśī* over the course of three days: Day-1, Day-2, and Day-3, which are indicated by the time interval between the instants of sunrise, i.e., t_1 to t_2, t_2 to t_3, and t_3 to t_4, respectively. Figure 20b depicts an exaggerated view of Day-1 in Figure 20a. This figure further depicts the division of the day into sixty *ghaṭikā*s and indicates the time duration of four *ghaṭikā*s (56–60) before sunrise as *aruṇodayakāla*.[194] Generally, people fast on the day when *ekādaśī* is observed at sunrise, i.e., on Day-2, and break the fast during the morning hours of the next day, i.e., on Day-3, strictly before the *dvādaśī* lapses. Śrī Madhvācārya, in his *Kṛṣṇāmṛtamahārṇava*, prescribes a general rule to postpone the *ekādaśī* fast.[195] According to this rule, even the *tithi* at sunrise is *ekādaśī*, if one observes the presence of *daśamī*-*tithi* during and before the *aruṇodayakāla*, as depicted in Figure 20c and Figure 20d respectively, one should avoid fasting on *ekādaśī* day, i.e., on Day-2, and observe it on *dvādaśī* day, i.e., on Day-3. This phenomenon of *ekādaśī* being hit (postponed) by *daśamī* is called *viddhaikādaśī*. The time period before sunrise, which is checked for the presence of *daśamī* for the occurrence of *viddhaikādaśī* is referred to as *daśamīvedhakāla*.

193 See *Ekādaśī-nirṇaya* verses 40–56, B. P. N. Rao (1994: 37–41). Also, see *Smṛtimuktāvalī*, Giri Ācārya (2013: 339–368), and *Śrī Vādirājara Kṛtigaḷu* composition 36, Nāgaratna (1980: 96).

194 See *Kṛṣṇāmṛtamahārṇava* verse 131(a,b), Bannañje (1974*a*: 91), and *Ekādaśī-nirṇaya* verse 3(a,b), B. P. N. Rao (1994: 25), which state चतस्रो घटिकाः प्रातररुणोदय उच्यते।

195 See *Kṛṣṇāmṛtamahārṇava* verse 129, Bannañje (1974*a*: 91). Also, see *Ekādaśī-nirṇaya* verse 2, B. P. N. Rao (1994: 25), which state अरुणोदयवेलायां दशमी यदि दृश्यते। पापमूलं तदा ज्ञेयम् एकादश्युपवासनम्॥ Further, see *Kṛṣṇāmṛtamahārṇava* verse 121(c,d), Bannañje (1974*a*: 90), which states उपोष्या द्वादशी पुण्या पूर्वविद्धां परित्यजेत्॥

Śrī Trivikramapaṇḍitācārya, through the above verse, clarifies Śrī Madhvācārya's general rule of *viddhaikādaśī* and provides specific time intervals before *aruṇodayakāla* for different types of *daśamī-tithi*. These time intervals denoted by '*x*' are given in the table within the Figure 20d. The types of *daśamī-tithi* are elucidated as follows.

Based on the duration of the *daśamī-tithi*, it is broadly classified into *ativṛddhi*, *samavṛddhi*, *sama*, and *hrāsa*. When the duration of *daśamī-tithi* exceeds 60 *ghaṭikā*s by either 5 or 6 *ghaṭikā*s, i.e., 65 or 66 *ghaṭikā*s, it is called *ativṛddhi*.[196] When the duration of *daśamī-tithi* exceeds 60 *ghaṭikās* by either 1, 2, or 3 *ghaṭikā*s, i.e., 61, 62, or 63 *ghaṭikā*s, it is reckoned as *samavṛddhi*.[197] When the duration of *daśamī-tithi* is 60 *ghaṭikā*s, it is called *sama*.[198] When the duration of *daśamī-tithi* is less than 60 *ghaṭikā*s, it is called *hrāsa*.[199]

In the case of *ativṛddhi*, 16 *liptis* (equivalent to 80 *vighaṭikā*s, or 1 *ghaṭikā* and 20 *vighaṭikā*s)[200] prior to *aruṇodayakāla* is checked for the presence of *daśamī*.[201] In the case of *samavṛddhi*, 1 *ghaṭikā* (equivalent to 12 *lipti*s) prior to *aruṇodayakāla* is checked for the presence of *daśamī*.[202] In the case of *sama* and *hrāsa*, 2 *lipti*s (equivalent to 10 *vighaṭikā*s) and 1 *lipti* (equivalent to 5 *vighaṭikā*s), respectively, prior to *aruṇodayakāla*, are checked for the presence of *daśamī*.[203] These four clas-

196 See *Ekādaśī-nirṇaya* verse 44(c,d), B. P. N. Rao (1994: 37–38), which states षड्दञ्चघटिकावृद्धिरतिवृद्धिरिहोच्यते॥ Bannañje (1974*b*: 193) gives an example of *ativṛddhi* as: the duration of *tithi*s, in *ghaṭikā*s, after sunrise in the three successive days Day-1, Day-2, and Day-3 are *daśamī* - 45 *ghaṭikā*s, *ekādaśī* - 50 *ghaṭikā*s, and *dvādaśī* - 55 *ghaṭikā*s, respectively. The duration of *ekādaśī-tithi* will be equal to the sum of the time durations of *ekādaśī* in Day-1 and Day-2, i.e., (15 + 50 =) 65 *ghaṭikā*s. As the duration of *tithi* will not vary significantly over two successive days, the duration of *daśamī-tithi* is ≈ 65 *ghaṭikā*s. Bannañje (1974*b*: 192) also assumes the duration of *tithi* with four *ghaṭikā*s excess of 60 *ghaṭikā*s, i.e., 64 *ghaṭikā*s, also as *ativṛddhi*.

197 See *Ekādaśī-nirṇaya* verse 45(a,b), B. P. N. Rao (1994: 37–38), which states एकद्वित्र्यात्मिका वृद्धिः समवृद्धिरिति स्मृता। Similar to *ativṛddhi*, Bannañje (1974*b*: 193) gives an example of *samavṛddhi* as: *daśamī* - 23 *ghaṭikā*s, *ekādaśī* - 24 *ghaṭikā*s, *dvādaśī* - 25 *ghaṭikā*s. It implies that the duration of *daśamī-tithi* is ≈ 61 *ghaṭikā*s.

198 Similarly, Bannañje (1974*b*: 193) gives an example of *sama* as: *daśamī* - 46.5 *ghaṭikā*s, *ekādaśī* - 47 *ghaṭikā*s, *dvādaśī* - 46.5 *ghaṭikā*s. It implies that the duration of *daśamī-tithi* is ≈ 60 *ghaṭikā*s.

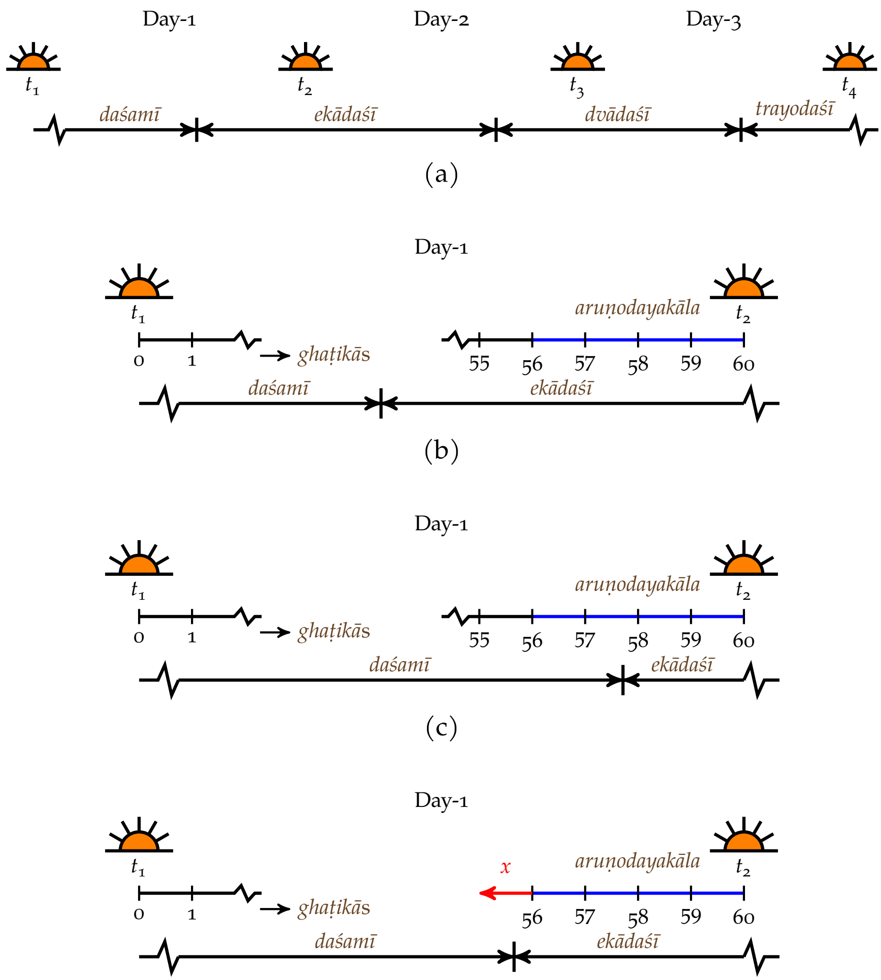
199 Similarly, Bannañje (1974*b*: 193) gives an example of *hrāsa* as: *daśamī* - 55 *ghaṭikā*s, *ekādaśī* - 50 *ghaṭikā*s, *dvādaśī* - 45 *ghaṭikā*s. It implies that the duration of *daśamī-tithi* is ≈ 55 *ghaṭikā*s.

200 Refer footnote 190. Also see, *Ekādaśī-nirṇaya* verses 47(c,d)–48(a,b), B. P. N. Rao (1994: 38), which state लिप्तिर्विघटिकाः पञ्च लिप्तयो द्वादशैव तु॥ घटिकैकेति विज्ञेया ज्योतिःशास्त्रप्रमाणतः।

201 See *Ekādaśī-nirṇaya* verses 53(c,d)–54(a,b), B. P. N. Rao (1994: 40), which state अतिवृद्धावष्टयुगमष्टद्वन्द्वं हि षोडश॥ लिप्तयो वेधहीनाः स्युः...।

202 See *Ekādaśī-nirṇaya* verses 54(c,d)–55, B. P. N. Rao (1994: 40), which state सति साम्ये न वृद्धिश्चेदिति यावत्तदष्टकम्॥ चतुष्टयं च लिप्तिनामेवं द्वादशलिप्तयः। घटिकैका भवेत्सर्वा दशमी वेधवर्जिता।

203 See *Ekādaśī-nirṇaya* verses 50(c,d)–51(a,b), B. P. N. Rao (1994: 39), which state समे द्विलिप्तिका मात्रं ह्रासे त्वेकैव सा मता॥ चतुष्कादुत्तरं त्वेतद्दशमी वेधवर्जनम्।



Daśamī tithi		'x' in
Type	*Duration in ghaṭikās*	*vighaṭikās*
hrāsa	< 60	5
sama	= 60	10
samavṛddhi	61, 62 or 63	60
ativṛddhi	65 or 66	80

(d)

Figure 20: (a) A diagram showing the transition of *tithis*, *daśamī*, *ekādaśī*, and *dvādaśī* across three days, (b,c and d) Diagrams showing the *tithi* transition from *daśamī* to *ekādaśī* without *daśamīvedha*, *daśamīvedha* in *aruṇodayakāla*, and *daśamīvedha* in x *vighaṭikās* before *aruṇodayakāla* with different values of x tabulated.

sifications of *viddhaikādaśī* depending on the type of *daśamī-tithi* are summarized in Figure 20d.

In conclusion, if one observes the presence of *daśamī-tithi* in the time interval of '4 *ghaṭikā*s + *x* *vighaṭikā*s' before sunrise, it is considered as *viddhaikādaśī*, prompting the postponement of the fast to the following day.

17 FASTING DAYS OF *VIṢṆUPAÑCAKA-VRATA*

उदयव्यापिनी दर्शा पौर्णमासी तु यामिका ।
मध्याह्नव्यापिनी श्रोणा उपोष्या विष्णुतत्परैः ॥ २६ ॥ ॥ अनुष्टुभ् ॥

udayavyāpinī darśā paurṇamāsī tu yāmikā |
madhyāhnavyāpinī śroṇā upoṣyā viṣṇutatparaiḥ || 26 || || *anuṣṭubh* ||

The new moon (*darśā*) prevailing in the morning (*udayavyāpinī*), the full moon (*paurṇamāsī*) lasting for a *yāma*, and the *Śravaṇā-nakṣatra* (*śroṇā*) prevailing in the afternoon (*madhyāhnavyāpinī*) are [considered] fast-worthy by the followers of Viṣṇu.

The above verse prescribes the days of a month on which the devotees of Viṣṇu shall observe fast. They are the days in which:

- the *amāvāsyā tithi* prevails in the morning (*prātaḥ-kāla*).[204]
- the *pūrṇimā tithi* prevails at least for the duration of a *yāma* (3 hours or 7.5 *ghaṭikā*s)[205] after sunrise.
- the *Śravaṇā nakṣatra* prevails till noon (*madhyāhna-kāla*) after sunrise.

Śrī Kṛṣṇācārya, in his *Smṛtimuktāvalī*, also provides a similar verse and attributes it to *Bhaviṣyat-purāṇa*.[206]

In common parlance, a *vrata* observed for Viṣṇu on the prescribed five days of a month — the above three days along with the two *ekādaśī*s — for the duration of a year ($12 \times 5 = 60$ fasting days) is called *Viṣṇupañcaka*.[207]

204 In *Smṛtimuktāvalī*, Śrī Kṛṣṇācārya states thus: Vedavyāsa divides the day (time between sunrise and sunset) into five parts; each equal to three *muhūrta*s. They are: *prātaḥ-kāla* (morning), *saṅgava-kāla*, *madhyāhna-kāla* (noon) , *aparāhṇa-kāla* (afternoon), and *sāyam-kāla* (evening) in order. See Giri Ācārya (2013: 250).

205 See Bannañje (1974*b*: 193), Monier-Williams (1986: 850).

206 Śrī Kṛṣṇācārya cites the verse: उदयव्यापको दर्शः पौर्णमासी तु यामिका। मध्याह्नव्यापिनी श्रोणा उपोष्या विष्णुतत्परैः॥ See Giri Ācārya (2013: 533).

207 See Giri Ācārya (2013: 533–536).

18 REAPING THE FULL BENEFITS OF A FAST

उपवासफलप्रेप्सुर्जह्याद् भुक्तचतुष्टयम् ।[208]
पूर्वापरे तु सायाह्ने सायम्प्रातस्तु मध्यमे ॥ २७ ॥ ॥ अनुष्टुभ् ॥

upavāsaphalaprepsurjahyād bhuktacatuṣṭayam |
pūrvāpare tu sāyāhne sāyamprātastu madhyame || 27 || || *anuṣṭubh* ||

One desirous of the benefits of the fast shall forego the quartet of meals — in the evenings of the previous and the next day [of the fast], and in the morning and evening of the middle (on the day of the fast).

The above verse prescribes the four meals that are to be foregone to enhance the benefits of a fast. These comprise the morning and evening meals on the day of the fast itself, along with the evening meals on the days preceding and following the fast. For instance, in the context of an *ekādaśī* fast, this means refraining from consuming the morning and evening meals on *ekādaśī*-*tithi*, as well as the evening meals on *daśamī* and *dvādaśī*. It is worth noting that a verse with similar instruction is found in *Skānda-purāṇa*.[209]

19 *SAṄKOCA-DVĀDAŚĪ* OR *SĀDHANA-DVĀDAŚĪ*

कलार्धं द्वादशीं दृष्ट्वा निशीथादूर्ध्वमेव तु ।
आमध्याह्नाः क्रियाः सर्वाः कर्तव्याः शम्भुशासनात् ॥ २८ ॥ ॥ अनुष्टुभ् ॥

kalārdhaṃ dvādaśīṃ dṛṣṭvā niśīthādūrdhvameva tu |
āmadhyāhnāḥ kriyāḥ sarvāḥ kartavyāḥ śambhuśāsanāt || 28 || || *anuṣṭubh* ||

Upon knowing [that] *dvādaśī* [lasts] for half of a *kalā* (*kalārdhaṃ*)[210] [after sunrise], all the rituals that have to be performed till noon are to be performed after midnight [of the previous day] as per the instruction of Śambhu.[211]

The above verse is excerpted from Śrī Madhvācārya's *Kṛṣṇāmṛtamahārṇava*.[212] It gives instructions on how to perform *dvādaśī vrata*, in the case where there are only a few minutes of *dvādaśī* left after sunrise.[213]

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208 Bannañje (1974*b*: 193) notes an alternate reading भक्तचतुष्टयम्।

209 See Karaṇam and Vādirājācārya (2002: 267), which notes
दशम्याश्चैव नक्तश्च एकादश्यामुपोषणम्। द्वादश्यामेकभुक्तं च अखण्डा इति कथ्यते॥ २.५.१२.२३ ॥

210 As 1 *kalā* = 1 *lipti*, from footnote 190, 1 *kalārdhaṃ* = 2.5 *vināḍikā*s. Also, See Vyāsadāsa (2007: 44), and Bannañje (1974*b*: 193).

211 Vyāsadāsa (2007: 44) interprets Śambhu as Caturmukha Brahma, while it is generally interpreted as Īśvara, Giri Ācārya (2013: 459).

212 See *Smṛtimuktāvalī*, Giri Ācārya (2013: 459).

213 See *Smṛtimuktāvalī*, Giri Ācārya (2013: 457–465).

A typical *dvādaśī vrata* is an act of consuming a meal in the morning of *dvādaśī tithi* within the stipulated time (strictly before the *dvādaśī* elapses) thus completing the *ekādaśī vrata* (fast). The consequences of transgressing *dvādaśī vrata* are elucidated in *Kṛṣṇāmṛtamahārṇava*.[214] Owing to all the consequences mentioned, how could one complete a meal (after performing all the daily rituals like *Sandhyāvandana*, etc.) if there are only a few minutes of *dvādaśī tithi* left after sunrise? This is the problem addressed by the above verse.

To avoid the violation of *dvādaśī vrata*, it is prescribed that all the activities that are usually to be performed till afternoon (like *Sandhyāvandana*, *Aupāsana*, etc., in the morning and *Devatārcana*, *Vaiśvadeva*, etc., in the afternoon) are to be completed before the sunrise by starting them from the midnight of the previous day. In common parlance, such a *dvādaśī* which remains a few minutes after sunrise, is known as *Saṅkoca-dvādaśī* or *Sādhana-dvādaśī*.[215]

20 DISCUSSION

THE TITHINIRṆAYA UNIQUELY COMPRISES both the procedure to compute the *tithi* at sunrise for an observer at a latitude of 12.78° and the religious injunctions for fasting days dedicated to Lord Viṣṇu. It features 28 verses composed in three different meters that enhance the beauty of the text. While most verses are set in the classical *anuṣṭubh* meter, the author occasionally employs the melodious *vaṃśastha* and *āryā* meters too. Remarkably, all *kaṭapayādi* phrases employed in the *Tithinirṇaya*, such as *māpati* (615), *murāri* (225), etc., can be interpreted as epithets of Viṣṇu. This appears to indicate the author's deep devotion towards this deity.

The astronomical portions of the *Tithinirṇaya* adhere to a typical *karaṇa* genre and adopt Haridatta's *parahita* modified *Āryabhaṭīya* parameters. As is common in the *karaṇa* genre, *Tithinirṇaya* provides a simple procedure for computing *tithi*, avoiding complex astronomical formulae by using interpolation for corrections such as *bhujāntara*, *manda*, and *caradala*.[216] It ignores the *udayāntara* correction, does not provide expressions for the true rates of motion of the Sun and the Moon, and even approximates the duration of the *tithi*, a varying quantity, to 60 *ghaṭikā*s. Since the procedure involves only basic mathematical operations, the text appears to be intended for laypeople who are not well-versed in astronomy.

In our work, we enhance the understanding of the procedures given in the *Tithinirṇaya* by stating the necessary assumptions, providing the mathematical

214 See *Kṛṣṇāmṛtamahārṇava* verses 157–159, Bannañje (1974*a*: 94).

215 Here, *saṅkoca* means 'prevailing for a short time,' and *sādhana* means 'to be achieved with great effort.' See Giri Ācārya (2013: 463).

216 The interpolated values of Rsine are utilized in *bhujāntara*, and *manda* corrections. The interpolated values of *cara* ($2\Delta\alpha$) are used in *caradala* correction.

and geometric rationales, and offering comments at each stage. We have elaborated on the correct sequence of corrections and discussed our disagreements with the interpretations of the commentators Govindācārya and Vyāsadāsa. Further, we have attempted to explain the concept of *viddhaikādaśī*, stated in verse 25, in the light of Śrī Vādirājatīrtha's *Ekādaśī-nirṇaya*. We have studied the question of the authorship of the *Tithinirṇaya* and argued that Śrī Trivikramapaṇḍitācārya could be the probable author of the text. We have discussed the similarities in the verses, expressions, and procedures between *Tithinirṇaya* and the astronomical texts such as *Grahacāranibandhanasaṅgraha*, *Laghubhāskarīya* and its commentaries, and *Karaṇaratna*, as well as the religious texts such as *Bhaviṣyat-purāṇa*, *Skānda-purāṇa*, and *Kṛṣṇāmṛtamahārṇava*.

In conclusion, the *Tithinirṇaya* is a simple handbook for computing the *tithi* at sunrise and determining the day on which *ekādaśī* fast must be observed. As the observance of the *ekādaśī* fast is a core tenet of the Mādhva tradition, *Tithinirṇaya* holds great significance for the community and remains in use in several *maṭha*s. However, it may be noted that, based on the *cara* values given in the text, this work appears to be intended only for observers located at a latitude (ϕ) of 12.78°. Additionally, the *parahita* system on which the *Tithinirṇaya* is based has been superseded by Parameśvara's *dṛggaṇita* system and others. Therefore, certain revisions are necessary to align computations with observations in the present day.

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APPENDIX A: SYMBOLS AND THEIR DESCRIPTION

Symbol	Description
A	*Kali-ahargaṇa*, or the civil days elapsed since the start of *kaliyuga* (*kalyādi*)
A'	Civil days elapsed since the epoch
D_c	Civil days in *mahāyuga*
S_y	*Śaka* years elapsed since *kalyādi*
K_y	*Kali* years elapsed since *kalyādi*
K_y^e	*Kali* years elapsed till epoch
K_y^{se}	*Kali* years elapsed since the epoch
g	Multiplier
h	Divisor
E_{tithi}	Number of *tithis* elapsed
$E_{ghaṭikā}$	*Ghaṭikās* elapsed in the current *tithi*
E_{lipti}	Additional *liptis* elapsed in the current *tithi*
Locations and measurements on the spherical Earth	
L	Laṅkā, or the point of intersection of prime meridian and equator
L'	Point of intersection of observer's meridian and equator
L'_E	L' with observer's meridian aligned to the east of prime meridian
L'_W	L' with observer's meridian aligned to the west of prime meridian
Q	Location of the observer
ϕ	Latitude of the observer (Q)
C	Circumference of the spherical Earth in *yojanas*
Δd	Distance between prime meridian and observer's meridian in *yojanas* along the equator (LL')
Δl	Longitudinal difference between observer's meridian and prime meridian
P_N	North Pole

continued ...

...continued

Symbol	Description
P_S	South Pole
Time instants and time differences	
t^k	Time instant of mean sunrise at Laṅkā (L) at *kalyādi*
t^e	Time instant of mean sunrise at Laṅkā (L) at epoch
t°	Time instant of mean sunrise at Laṅkā (L) on *kali-ahargaṇa* A
t^d	Time instant of mean sunrise at L'
t^b	Time instant of true sunrise at L'
t^u	Time instant of true sunrise at L' considering the obliquity of the ecliptic
t^{ca}	Time instant of true sunrise at Q
Δt^d	Time difference ($t^d \sim t^\circ$) between the instants of mean sunrise at L' and L
Δt^b	Time difference ($t^b \sim t^d$) between the instants of true and mean sunrise at L'
Δt^u	Time difference ($t^u \sim t^b$) between the instants of true sunrise at L' with and without considering the obliquity of the ecliptic
Δt^{ca}	Time difference ($t^{ca} \sim t^u$) between the instants of true sunrise at Q and L'
Elements of Celestial Sphere	
Z_L	Zenith of the observer at Laṅkā (L)
$Z_{L'}$	Zenith of the observer at L'
Z_Q	Zenith of the observer at Q
E	East cardinal point
W	West cardinal point
PM	Prime Meridian
S	Sun
S_d	*Deśāntara* corrected Sun

continued ...

...continued

Symbol	Description
S_b	*Bhujāntara* corrected Sun
S_u	*Udayāntara* corrected Sun
S^{-90}	A fictitious point object 90° behind the Sun
S_d^{-90}	A fictitious point object 90° behind the *deśāntara* corrected Sun (S_d)
S_b^{-90}	A fictitious point object 90° behind the *bhujāntara* corrected Sun (S_b)
DoV	Direction of View
U	Apogee
M	Starting point of the sign *Meṣa*(*meṣādi*)
R	Radius of the orbit
r (or) r_d	Eccentricity of the orbit
K (or) K_d	*Manda*-*karṇa*, or the true distance of the planet
k	*Kendra*, or anomaly of the planet
b	*Bhuja* of an arc
Γ	Vernal equinox
Ω	Autumnal equinox
ϵ	Obliquity of the ecliptic
δ_s	Declination of the Sun
α_s	Right ascension of the Sun
θ_Γ	Motion of vernal equinox
C_Γ	A fictitious celestial body on the ecliptic moving opposite to the direction of the Sun with the time period of revolution equal to the oscillation time period of equinox
$\bar{A}$	*Ayana* or the *sāyana* longitude (measured clockwise) of the fictitious celestial body C_Γ
I	Point on the 6 o'clock circle indicating the point of sunrise for an observer (L') on the equator

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Symbol	Description
J	Point on the 6 o'clock circle indicating the point of sunset for an observer (L') on the equator
X	Point of sunrise for an observer (Q) situated in the northern latitude
Y	Point of sunset for an observer (Q) situated in the northern latitude
$\Delta\alpha$	Ascensional difference
Revolutions	
R_s	Revolutions of the Sun in *mahāyuga*
R_m	Revolutions of the Moon in *mahāyuga*
R_{m_ap}	Revolutions of the Moon's apogee in *mahāyuga*
R_m^c	Corrected revolutions of the Moon in *mahāyuga* due to *parahita*
$R_{m\_ap}^c$	Corrected revolutions of the Moon's apogee in *mahāyuga* due to *parahita*
Rates of Motion	
$\dot{\theta}_p^{\circ}$	Mean rate of motion of the planet (p)
$\dot{\theta}_s^{\circ}$	Mean rate of motion of the Sun
$\dot{\theta}_m^{\circ}$	Mean rate of motion of the Moon
$\dot{\theta}_{m\_ap}^{\circ}$	Mean rate of motion of the Moon's apogee
$\dot{\theta}_m^c$	Corrected mean rate of motion of the Moon due to *parahita*
$\dot{\theta}_{m\_ap}^c$	Corrected mean rate of motion of the Moon's apogee due to *parahita*
$\dot{\theta}_p^t$	True rate of motion of the planet (p)
$\dot{\theta}_s^t$	True rate of motion of the Sun
$\dot{\theta}_m^t$	True rate of motion of the Moon
Corrections	

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Symbol	Description
Δ_p°	*Parahita* correction for the mean longitude of the planet *p*
$\dot{\Delta}_p^\circ$	*Parahita* correction for the mean rate of motion of the planet *p*
$\dot{\Delta}_m^\circ$	*Parahita* correction for the mean rate of motion of the Moon
$\dot{\Delta}_{m\_ap}^\circ$	*Parahita* correction for the mean rate of motion of the Moon's apogee
Δ_p^d	*Deśāntara* correction of the planet (p)
Δ_s^d	*Deśāntara* correction of the Sun
Δ_m^d	*Deśāntara* correction of the Moon
$\Delta_{m\_ap}^d$	*Deśāntara* correction of the Moon's apogee
${}^d\Delta_s^m$	*Manda* correction of the *deśāntara* corrected Sun (S_d)
${}^d\Delta_s^b$	Mean *bhujāntara* correction of the *deśāntara* corrected Sun (S_d)
${}^d\Delta_m^b$	Mean *bhujāntara* correction of the *deśāntara* corrected Moon
${}^m\Delta_s^b$	True *bhujāntara* correction of the *manda* corrected Sun
${}^b\Delta_s^m$	*Manda* correction of the *bhujāntara* corrected Sun (S_b)
${}^b\Delta_m^m$	*Manda* correction of the *bhujāntara* corrected Moon
Δ_p^u	*Udayāntara* correction of the celestial body *p*
Δ_p^{ca}	*Caradala* correction of the celestial body *p*
Δ_s^{ca}	*Caradala* correction of the Sun
Δ_m^{ca}	*Caradala* correction of the Moon
Longitudes	
θ_s^k	Mean longitude of the Sun at the instant (t^k) of mean sunrise at Laṅkā (L) at *kalyādi*
θ_m^k	Mean longitude of the Moon at the instant (t^k) of mean sunrise at Laṅkā (L) at *kalyādi*
$\theta_{m\_ap}^k$	Mean longitude of the Moon's apogee at the instant (t^k) of mean sunrise at Laṅkā (L) at *kalyādi*

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Symbol	Description
θ_m^{ck}	Corrected mean longitude of the Moon at the instant (t^k) of mean sunrise at Laṅkā (L) at *kalyādi*
$\theta_{m\_ap}^{ck}$	Corrected mean longitude of the Moon's apogee at the instant (t^k) of mean sunrise at Laṅkā (L) at *kalyādi*
θ_s^e	Mean longitude of the Sun at the instant (t^e) of mean sunrise at Laṅkā (L) at epoch
θ_m^e	Mean longitude of the Moon at the instant (t^e) of mean sunrise at Laṅkā (L) at epoch
$\theta_{m\_ap}^e$	Mean longitude of the Moon's apogee at the instant (t^e) of mean sunrise at Laṅkā (L) at epoch
θ_p°	Mean longitude of the planet (p) at the instant (t°) of mean sunrise at Laṅkā (L) for *kali-ahargaṇa* A
θ_s°	Mean longitude of the Sun at the instant (t°) of mean sunrise at Laṅkā (L) for *kali-ahargaṇa* A
θ_m°	Mean longitude of the Moon at the instant (t°) of mean sunrise at Laṅkā (L) for *kali-ahargaṇa* A
$\theta_{m\_ap}^\circ$	Mean longitude of the Moon's apogee at the instant (t°) of mean sunrise at Laṅkā (L) for *kali-ahargaṇa* A
θ_{s_ap}	Fixed longitude of the Sun's apogee
λ_s	*Sāyana* longitude of the Sun
Resulting longitudes due to corrections	
θ_p^d	Mean longitude of the planet (p) at the instant (t^d) of mean sunrise at L'
θ_s^d	Mean longitude of the Sun at the instant (t^d) of mean sunrise at L'
θ_m^d	Mean longitude of the Moon at the instant (t^d) of mean sunrise at L'
${}^d\theta_s^b$	Mean longitude of the Sun at the instant (t^b) of true sunrise at L'
${}^d\theta_m^b$	Mean longitude of the Moon at the instant (t^b) of true sunrise at L'

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Symbol	Description
${}^{d}\theta_s^m$	True longitude of the Sun at the instant (t^d) of mean sunrise at L'
${}^{d}\theta_m^m$	True longitude of the Moon at the instant (t^d) of mean sunrise at L'
${}^{b}\theta_p^m$	True longitude of the planet (p) at the instant (t^b) of true sunrise at L'
${}^{b}\theta_s^m$ (${}^{m}\theta_s^b$)	True longitude of the Sun at the instant (t^b) of true sunrise at L'
${}^{b}\theta_m^m$ (${}^{m}\theta_m^b$)	True longitude of the Moon at the instant (t^b) of true sunrise at L'
θ_p^u	True longitude of the planet (p) at the instant (t^u) of true sunrise at L' considering the obliquity of the ecliptic
θ_s^u	True longitude of the Sun at the instant (t^u) of true sunrise at L' considering the obliquity of the ecliptic
θ_m^u	True longitude of the Moon at the instant (t^u) of true sunrise at L' considering the obliquity of the ecliptic
θ_p^t	True longitude of the planet (p) at the instant (t^{ca}) of true sunrise at Q
θ_s^t	True longitude of the Sun at the instant (t^{ca}) of true sunrise at Q
θ_m^t	True longitude of the Moon at the instant (t^{ca}) of true sunrise at Q

APPENDIX B: PHRASE, ITS NUMBER, AND THE CORRESPONDING MEANING

Phrase	Number	Meaning
Section 3: Verses 2 – 3		
bhūśrī-bhinnāki-cintya	1610424	Contemplated [also] by heavenly (liberated) souls who are different [in nature] from Bhūdevī and Śrīdevī (forms of Mahālakṣmī)
garuḍa-dhyeya	11323	One who is meditated upon by Garuḍa (vehicle of Viṣṇu; eagle)
dhīsūnu-nāga	30079	From whose mind the *śāstra*s spring forth
deśādhāra-harārpakam	11; 28, 29, 58	One who imparts the sustenance and dissolution of the Earth
kāla	31	One who controls Time
go	3	One who is omniscient
Section 4: Verse 4		
ananta	600	Infinite in terms of attributes (knowledge, compassion, etc.), time and space
baudhāṅga-tulya	16393	One who treats the knowledgeable ones as his own
śuka	15	One who shines magnificently
prājñāñjalibhṛd	43802	One who receives the prayers of the learned, or one who holds the *jñāna-mudrā*
tāraśobhā-tinākinī	01; 06, 45, 26	One who surpasses the splendor of stars in the heaven (female form of Viṣṇu, Mohinī)
Section 5: Verse 5		
drāgarāgā	3232	One who instantly bestows the virtue of non-attachment to worldly things (*vairāgya*)
nibhā	40	[One who has] splendor
jagat-senāṅga	30738	One who commands the army in the world

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Phrase	Number	Meaning
śreṣṭha-cintyo'mbun-ā'rcane	06; 03, 16, 22	While being worshiped with consecrated waters by the demigods (*śreṣṭha*), He is [the One who is] contemplated upon
Section 6: Verses 6 – 7		
pāpa	11	One who delivers sins to the people according to their deeds
arka	10	One who is worshiped
anarka	100	One who is worshiped by Vāyu
sānubhū	407	One who is complete [without blemishes] and [hence] followed by Mahālakṣmī
Section 7: Verses 8 – 9		
divya	18	One who is heavenly, not made up of earthly matter, one who is playful
praja	82	One who sustains the people
duṣṭāstrī	2; 18, 0, 0	One who disfigured the demoness Śūrpaṇakhā
Section 8: Verses 10 – 12		
śarīranut	225	Instigator of beings [into action etc.]
dhībhavana	449	The abode of intellect
kathañcana	671	The cause of extraordinary events
naḷījana	890	One by whom people are bound [to the cycle of birth and death]
mānapaṭu	1105	Skilled in epistemology
śukālapa	1315	One who talks [sweetly] like a parrot
nirāmaya	1520	Without any diseases or blemishes
dhīḥpathika	1719	One who is attained by the path of knowledge
nṛpādhika	1910	Protector of humans [and other beings] while being superior to them

continued ...

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Phrase	Number	Meaning
budhonara	2093	One who makes the intellectually weak also shine [with intelligence]
suptakhara	2267	One by whom Khara is put to sleep (slayer of the demon Khara)
kalāvirāṭ	2431	Supreme manifestation of arts
mahāśara	2585	One who has a powerful bow (*Śārṅga*)
dūrasara	2728	One who moves away [from the impious]
dhamīhari	2859	One who has the Vedas as means to attain Him and One who removes our sins
hasandhara	2978	One who sustains [the world] with a smile
vedanaga	3084	One who has Vedas as his stage
susaṅkula	3177	Melting pot of all good things
tamaḥkhaga	3256	One who impels the sense-organs and the intellect of people
pārabala	3321	Zenith of power
rasābala	3372	The essence of great strength
dhanāvali	3409	One who has heaps of wealth
kālabhṛgu	3431	One who reckons and moves the [wheels] of Time
jagadbhaga	3438	One who bestows prosperity to the world, or one who is most prosperous [and therefore to be attained] in the world
Section 9: Verse 13		
śubhāṅga	$3^{\circ}45'$	One with beautiful limbs
śubhrāgra	225	One who is pure and supreme
murāri	225	Kṛṣṇa, the enemy [slayer] of [the demon] Mura
Section 11: Verse 15		
sad	7	Blemish-less; one who liberates beings from the cycle of birth and death

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Phrase	Number	Meaning
aja	80	One without a birth; eternal
Section 12: Verses 16 – 18		
dhenubhava	4409	One who resides in cows
māpati	615	Husband of Mahālakṣmī
prabhāratna	242	Ultimate light
dhīsavana	479	One who is attained by knowledge-ritual (by the act of obtaining His knowledge)
gānasthāna	703	One who is the subject of music
janedhana	908	One who bestows prosperity to people
dehinitya	1088	One whose body is eternal
sugaprāya	1237	One whom the excellent seers reach forever
sāvalokya	1347	One who is known by the scriptures
taṭidvapu	1416	One whose body has the splendor of lightning
navabhāryā	1440	One whose wife (Mahālakṣmī) is newly-wed and is eternal
jño'nanta	600	One who is omniscient and infinite in terms of attributes (knowledge, compassion, etc.), time, and space
Section 14: Verses 19 – 22		
arkapūjya	1110	One who is worshipped by the Sun, and also by the use of *arka*-leaf (Calotropis gigantea)
sudhākara	2197	The repository and distributor of nectar
ratikrīḍa	3262	One who is playfully engaged [with the Gopikās]
nutaprabhu	4260	One to whom the lords bow
alaṅkṛṣṇa	5130	One who likes to get decorated
hitoddeśa	5868	One who has the welfare [of the all beings] as His objective

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Phrase	Number	Meaning
gatibhūta	6463	One who is the destination and the means of attainment [of pure bliss]
smarārdita	6825	One who makes people get afflicted by the God of Love
śaśidhāta	6955	The sustainer of Moon
anantāṅga	3600	One who has infinite organs (one who is infinite)

GLOSSARY

amāvāsyā The fifteenth *tithi* of a dark fortnight. 51, 129
aruṇodayakāla The time duration of four *ghaṭikās* before sunrise. 125–128

bhuja The angle traversed and yet to be traversed in the odd and even quadrants respectively. 56, 99–101, 106, 136

Caitra The first month of a lunar year. 53
cara Twice the ascensional difference ($2\Delta\alpha$) in *vighaṭikās*, is the time difference in the length of the day between the observers at latitude (ϕ) and equator. 55, 56, 59, 61, 105, 113–116, 118, 131, 132, 146
caradala Ascensional difference ($\Delta\alpha$) or half of the *cara*. 114, 118, 119, 122, 131

daśamī The tenth *tithi* of a bright or dark fortnight. 125–130, 146, 148
daśamīvedhakāla The time period before sunrise, which is checked for the presence of *daśamī* for the occurrence of *viddhaikādaśī*. 126
dhruva The fixed mean longitudes proposed by the author at the epoch. 50, 64, 68–70, 72–76
dṛgganita Computation of the positions and motions of the celestial objects in line with the observation. 132
dvādaśī The twelfth *tithi* of a bright or dark fortnight. 126–128, 130, 131

ekādaśī The eleventh *tithi* of a bright or dark fortnight. 51, 55, 61, 62, 125–132, 148

gata-jyā Elapsed Rsine. 98
ghaṭikā A time unit indicating the sixtieth part of a mean civil day. 78, 114, 123–129, 131, 134, 146–148
grahabhramaṇavṛtta The orbit of a planet. 102
guṇakāra A multiplier. 71
gurvakṣara One-sixtieth of a *vināḍikā*, or time it takes for a healthy person to pronounce a long syllable. 114, 115

hāraka A divisor. 71

iṣṭa-jyā Desired Rsine. 98

jyā The semi-chord of a semi-arc.. 98
jyārdha . 98, *see jyā*

kakṣyāmaṇḍala An imaginary orbit situated at the Earth's center and sharing the identical radius as the *pratimaṇḍala*. 86, 103
kalā A minute in arc units, or one-sixtieth of a degree. 64, 69, 73, 102, 113, 115, 130
kalā-śeṣa Remaining minutes in arc units. 98
kali-ahargaṇa The number of civil days elapsed since the beginning of the *kaliyuga*. 53, 64, 66, 67, 69, 70, 73, 74, 122, 134, 135, 139
kaliyuga The last quarter of a *mahāyuga* (4320000 years); the other three quarters being *kṛtayuga*, *tretāyuga*, and *dvāparayuga*. 53, 65, 72, 107, 134, 146, 147
kalyādi The start of *kaliyuga*. 59, 65–68, 70, 72–75, 107, 134, 135, 138, 139

karaṇa A genre of astronomical texts that chooses a recent epoch and dictates a simpler procedure in computing the aspects of astronomy, i.e., calendrical elements, eclipses, etc., without presenting the rationale involved in the computations. 50, 53, 68, 70, 74, 131

karaṇa Half of the duration of *tithi*. 52, 55, 147

karṇa Hypotenuse of a right-angled triangle. 85, 103, 136

kendra Also known as anomaly, which is the difference between the longitude of the planet and its apogee. 92, 93, 95, 98, 101, 102, 136

Kīlaka The forty-second year in a sixty-year cycle. 53

Laṅkā A location on the Earth where the prime meridian (a meridian passing through Ujjayinī, Svāmīnagara, etc.) intersects the equator. 56–59, 64–70, 72–74, 76–78, 82, 134, 135, 138, 139

lipti . 76, 77, 83, 113, 123–127, 130, 134, *see kalā*

Mādhva Related to Śrī Madhvācārya. 50, 51, 53, 55, 62, 118, 132

Mādhvas Followers of Śrī Madhvācārya. 51

mahāyuga A time cycle corresponding to 4320000 years, which comprises of *kṛtayuga*, *tretāyuga*, *dvāparayuga*, and *kaliyuga*. 59, 60, 67, 70, 71, 73, 75, 94, 134, 137, 146

maṭha A religious establishment in the lineage. 50–52, 54, 55, 132, 133

meṣādi The starting point of Aries (*Meṣa*), or 0° point in the Zodiac (*rāśicakra*). 65, 66, 82, 84, 85, 88, 91, 92, 99, 103, 105, 107–109, 136, 147

Meṣa-saṅkrānti The Sun's transition from Pisces (*Mīna*) to Aries (*Meṣa*) in the Zodiac (*rāśicakra*). 53

muhūrta Time period equal to twice a *ghaṭikā*. 129

nāḍikā . 78, 123–125, *see ghaṭikā*

nakṣatra Twenty seventh part (13°20′) of the ecliptic. 51, 52, 55, 129, 147, 148

nirayana The longitude of a celestial body measured with respect to *meṣādi*. 105–107, 110, 114

pañcāṅga An Indian calendar, which comprises of five elements: *tithi*, *vāra*, *nakṣatra*, *yoga*, and *karaṇa*. 51, 55

parahita A system proposed by Haridatta to correct the longitudes of the planets, computed from *Āryabhaṭīya* astronomical parameters, post *śaka* 444 or *kali* year 3623. 50, 63, 70–72, 75, 131, 132, 137, 138, 148

pratimaṇḍala . 102, 103, 146, *see grahabhramaṇavṛtta*

pūrṇimā The fifteenth *tithi* of a bright fortnight. 51, 129

rāśi One-twelfth part (30°) of the ecliptic, or a sign in the Zodiac (*rāśicakra*). 59, 64, 68–70, 73, 74, 95, 99, 101, 105, 108

rāśicakra Zodiac. 99, 100, 147

Sarvamūlagrantha A collection of 37 works attributed to Śrī Madhvācārya. 51

sāyana The longitude of a celestial body measured with respect to the vernal equinox. 105–108, 110, 114, 115, 119, 136

Śālivāhana-śaka The epoch corresponding to the (elapsed) 3179 years of *kaliyuga*. 50, 53, 71, 72, 134, 147, 148

śakābdasaṃskāra A correction, in *parahita* system of Haridatta, to correct the longitudes of the planets post *śaka* 444. 70, 72, 74
śiṣṭa-vartamānajyā Current Rsine difference. 98
Śravaṇā The twenty-second *nakṣatra*. 51, 129
śukla-caturthī The fourth *tithi* of a bright fortnight. 53
siddhānta A foundational treatise. 53

tithi Lunar day, or a time unit in which the longitudinal separation between the Moon and the Sun increases by 12°. 50, 51, 53–56, 62, 118, 123–132, 134, 146–148
Tithinirṇaya Determination of *tithi*. 50–57, 59–63, 70, 71, 80, 89, 90, 94, 97, 99, 105, 107, 109, 114, 115, 118, 122, 131–133
trayodaśī The thirteenth *tithi* of a bright or dark fortnight. 126, 128

vākya A word or a phrase that corresponds to a number. 95, 105, 113
vāra Weekday. 55, 147
viddhaikādaśī The *ekādaśī* which is being hit (postponed) by *daśamī*. 54–56, 63, 126, 127, 129, 132, 146
vighaṭikā A time unit indicating the sixtieth part of *ghaṭikā*. 78, 114–116, 118, 119, 121, 124, 125, 127–129, 146
vilipti A second in arc units: one-sixtieth of a minute, or one-three-thousand-six-hundredth (1/3600) of a degree. 76, 77, 83, 113, 114
vināḍikā . 78, 130, 146, *see vighaṭikā*
vrata A holy practice or ritual. 51, 56, 62, 129–131

yāma One-eighth part of a day, or a period of three hours. 129
yoga A time unit in which the sum of the longitudes of the Sun and the Moon increases by 800′. 52, 55, 147
yojana A unit of length used by Indian astronomers. 58, 76–78, 80, 82, 134

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